Identification and Infiltration in Consensus-type Networks

Airlie Chapman¹, Marzieh Nabi-Abdolyousefi¹, Mehran Mesbahi¹

¹U of Washington, Department of Aeronautics and Astronautics, (airliec, mnabi, mesbahi@u.washington.edu)

Problem Outline

Examination of a controlled network multi-agent system, operating with a consensus type algorithm that is under the influence of attached nodes.

- Introduction of an identification scheme, involving excitation and observation of the network by attached nodes.
- Identification of the spectrum of the underlying system matrix (modified graph Laplacian and flagging susceptible networks).
- Infiltration of these networks with the objective to sabotage by delivering a constant mean control signal.

Model and Example

\[
\dot{x}(t) = A(G, R)x(t) + B(R)u(t)
\]

\[
y(t) = C(\mathcal{F})x(t)
\]

- \(R = (R, E_R), \mathcal{F} = (F, E_F)\): Graph of excitation and observing nodes
- \(L(G)\): Graph Laplacian matrix
- \(A(G, R) = -(L(G) + M(R))\):
  - Modified Laplacian
  - \(B(R) = [B(R)]_{ij} = 1\) for \(\{r_i, v_j\} \in E_R\) and 0 otherwise
- \(M(R) = BB^T\)

Identification and Infiltration

Identification of a realization of the original system

\[
\left(\bar{A}, \bar{B}, \bar{C}, 0\right)
\]

Identifiable features:
- \(\bar{C}\bar{A}\bar{B} = C(\mathcal{F})A(G, R)B(R)\)
- The number of edges
- The number of disconnected components
- An accurate spectrum of the modified Laplacian.

Infiltration convergence cost

\[
J(x_0) = 2 \int_0^\infty x(t)^T x(t) \, dt = -x_0^T A(G, R)^{-1} x_0
\]

The security of the network may be qualified by:

- \(J_{\text{min}}(G, R) = \inf_{x_0} J(x_0)\)
- \(J_{\text{max}}(G, R) = \sup_{x_0} J(x_0)\)
- \(J_{\text{avg}}(G, R) = \mathbb{E}_{x_0} J(x_0)\)

\[
\frac{1}{n} \sum_{i=1}^{n} 1/\lambda_i(-A(G, R))
\]

Identified spectrum used to bound network security, flagging susceptible networks.

Theoretic Bounds

Analytic form of convergence for key graphs that bound all infiltrated graphs.

Demonstration

Two random graphs \(G_1^{10}\) and \(G_2^{10}\), each graph is identifiable from their solid nodes.

- \(0.10 \leq J_{\text{min}}(G_1^{10}) \leq 0.14\)
- \(0.15 \leq J_{\text{min}}(G_2^{10}) \leq 0.19\)
- \(0.34 \leq J_{\text{avg}}(G_1^{10})\)
- \(0.47 \leq J_{\text{avg}}(G_2^{10})\)
- \(0.54 \leq J_{\text{max}}(G_1^{10})\)
- \(0.69 \leq J_{\text{max}}(G_2^{10})\)

Conclusion

Investigated:
- Probing a network with limited initial knowledge through system identification techniques.
- Forming analytic bounds on the cost of infiltration via the system realization acquired by identification.
- Reasoning about the security of coordination algorithms.
- Identifying critical graph-theoretic parameters that can influence the synthesis of secured network geometries.