Semi-Autonomous Networks: Theory and Decentralized Protocols

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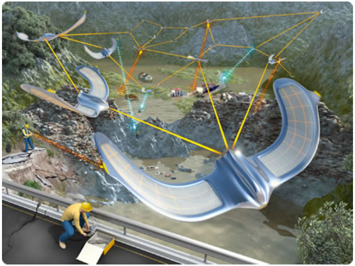
Motivation and Approach

- Semi-autonomous systems: adaption of consensus-type systems by introducing leader or influencing agents.
- How does network structure effect the efficiency of semi-autonomous systems?
- Can you dynamically adapt the network to encourage/deter the effect of influencing leaders/agents?



- Formulate the problem as an input-output dynamic system
- Relate system-theoretic metrics to the underlying network structure
- Form local edge swap protocols to increase/decrease these metrics

Graphs and Consensus Model

- Graph structure encapsulated by $\mathcal{G} = (V, E)$, where $V \in \mathbb{R}^n$ and $E \in \mathbb{R}^e$
- Matrix representation $L(\mathcal{G}) \in \mathbb{R}^{n \times n}$ where

$$[L(\mathcal{G})]_{ij} = \begin{cases} d_i & i = j \\ -1 & \{i,j\} \in E \\ 0 & otherwise \end{cases}$$

and d_i is the degree of node v_i.

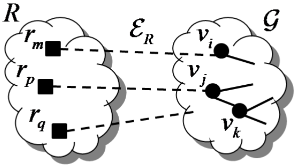
Consensus Model

$$\dot{x}_i(t) = \sum_{\{i,j\} \in E} (x_j(t) - x_i(t)) \iff \dot{x}(t) = -L(\mathcal{G})x(t)$$

- Studied in detail e.g. Jadbabaie '03, Olfati-Saber '07

Influenced Consensus Model

- Influencing node set
  $\mathcal{R} = (R, \mathcal{E}_R)$, $|\mathcal{E}_R| = r$
- $B(\mathcal{R}) \in \mathbb{R}^{n \times r}$, $[B(\mathcal{R})]_{ij} = 1$
  for $\{v_i, r_j\} \in \mathcal{E}_R$, 0 otherwise
- Dirichlet Matrix
  $-A(\mathcal{G}, \mathcal{R}) =$
  $L(\mathcal{G}) + B(\mathcal{R})B(\mathcal{R})^T$

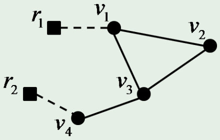


Influence Model

$$\dot{x}(t) = A(\mathcal{G}, \mathcal{R})x(t) + B(\mathcal{R})u(t)$$

Example:

$$A(\mathcal{G}, \mathcal{R}) = \begin{bmatrix} -3 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -3 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix},$$



$$B(\mathcal{R}) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Test Signal and Metrics

- Test signal: Unit intensity noise with mean u_c
- Mean Metric: Consider infinite time horizon convergence with $\tilde{x}(t) = x(t) - u_c \mathbf{1}$,

$$J(\mathcal{G}, \mathcal{R}, \tilde{x}(0)) = 2\int_0^\infty \tilde{x}(t)^T \tilde{x}(t) dt = -\tilde{x}(0)^T A(\mathcal{G}, \mathcal{R})^{-1} \tilde{x}(0).$$

The average $\mathbf{E}(\tilde{\mathrm{x}})$ convergence rate over $\|\tilde{x}(0) = 1\|$

$$J^{\mathrm{avg}}(\mathcal{G}, \mathcal{R}) = \frac{1}{n}\sum_{i=1}^{n} \left[-A(\mathcal{G}, \mathcal{R})^{-1}\right]_{ii}$$

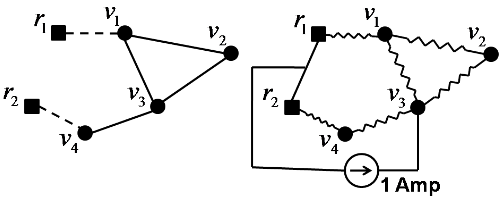
- Variance Metric: Variance of the states as $t \to \infty$ due to test signal is the trace of the controllability gramian $P(\mathcal{G}, \mathcal{R})$.

The average $\mathbf{Var}(\mathrm{x})$ at steady state

$$\mathbf{tr}(2P(\mathcal{G}, \mathcal{R})) = \frac{1}{n}\sum_{v_i \in \pi(\mathcal{E}_R)} \left[-A(\mathcal{G}, \mathcal{R})^{-1}\right]_{ii}$$

Effective Resistance of a Graph

- Consider edges $\mathcal{E}$ and $\mathcal{E}_R$ in the graph model replaced with 1Ω resistors, and nodes R shorted as a common node r_0.
- $\left[-A(\mathcal{G},\mathcal{R})^{-1}\right]_{ii}$ corresponds to node v_i's effective resistance to r_0, denoted $E_{\text{eff}}(v_i)$. [Barooah and Hespanha 2006]



Average $\mathbf{E}(\tilde{x})$

$$J^{\text{avg}}(\mathcal{G},\mathcal{R}) = \frac{1}{n}\sum_{i=1}^{n} E_{\text{eff}}(v_i)$$

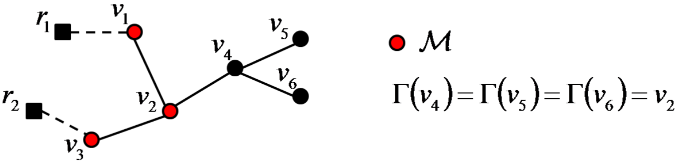
Average $\mathbf{Var}(x)$

$$\mathbf{tr}(2P(\mathcal{G},\mathcal{R})) = \frac{1}{n}\sum_{v_i\in\pi(\mathcal{E}_R)} E_{\text{eff}}(v_i)$$

Results over Trees $\mathcal{T}$

Finding E_{eff} is relatively simple over tree graphs $\mathcal{T}$

- Main path agents $\mathcal{M}$: Set of agents that lies on any shortest paths between agents in $\mathcal{R}$
- Subgraph $\mathcal{G}_{\mathcal{M}} = (\mathcal{M}, E_{\mathcal{M}})$: Agents $\mathcal{M}$ and edges between them
- Main path neighbor $\Gamma(v_i)$: Closest agent to v_i that is in $\mathcal{M}$



Average $\mathbf{E}(\tilde{\mathrm{x}})$ for trees

$$J^{\text{avg}}(\mathcal{T},\mathcal{R}) = \frac{1}{n}\left(\sum_{v_i\in\mathcal{M}} E_{\text{eff}}(v_i) + \sum_{v_i\notin\mathcal{M}}\left[E_{\text{eff}}(\Gamma(v_i)) + d(v_i,\Gamma(v_i))\right]\right)$$

Average $\mathbf{Var}(\mathrm{x})$ for trees

$$\mathbf{tr}(2P(\mathcal{T},\mathcal{R})) = \frac{|\mathcal{M}|}{n}\mathbf{tr}(2P(\mathcal{G}_{\mathcal{M}},\mathcal{R})).$$

Results over Trees and One Attached Agent $(\mathcal{T}, \mathcal{R}^i)$

Centrality Lemma

$$J^{\text{avg}}\left(\mathcal{T}, \mathcal{R}^i\right) = \frac{1}{n} \sum_{j=1}^{n} d\left(v_i, v_j\right) + 1$$

Single Bounds

$$2 - \frac{1}{n} \leq J^{\text{avg}}(\mathcal{T}, \mathcal{R}^i) \leq \frac{1}{2}(n+1)$$

Topology Independence

$$\mathbf{tr}(2P\left(\mathcal{G}, \mathcal{R}^i\right)) = \frac{1}{n}$$

Link to more general $\mathcal{G}$

- Rayleigh's Monotonicity Principle:
  "If the edge resistance in a electrical network is decreased then the effective resistance between any two agents in the network can only decrease."

Graphs and their underlying trees

For a graph $\mathcal{G}$ any underlying tree $\mathcal{T}$ has the property $J^{\text{avg}}(\mathcal{G},\mathcal{R}) \leq J^{\text{avg}}(\mathcal{T},\mathcal{R})$ and $\mathbf{tr}(P(\mathcal{G},\mathcal{R})) \leq \mathbf{tr}(P(\mathcal{T},\mathcal{R}))$.

Random Graph

For a random graph $\mathcal{G}_{n,p}$, almost surely, $1 < J^{\text{avg}}(\mathcal{G}_{n,p},\mathcal{R}^i) < 3$.

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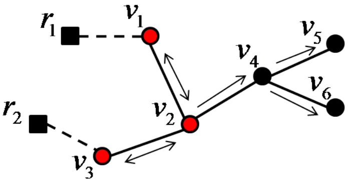
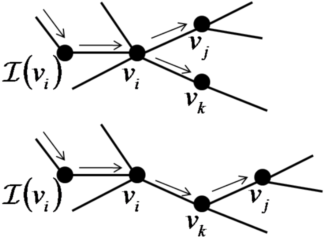
Adaptive Tree Protocols

- Local edge trades to deter/encourage the influence of attached agents
- Approach for all v_i:
  - If $v_i \in \pi(\mathcal{E}_R)$, broadcasts a detection signal (which are rebroadcasted $\forall v_i \in V$)
  - Forms a proximity set $\mathcal{I}(v_i)$, where $v_j \in \mathcal{I}(v_i) \implies$ Neighbor $v_j \in \mathcal{N}(v_i)$ is closer to a $r_i \in R$ than v_i.
  - Runs a local edge trade protocol with only knowledge of $\mathcal{I}(v_i)$.
- Node v_i is an element of the main path set $\mathcal{M}$, if $|\mathcal{I}(v_i)| > 1$.

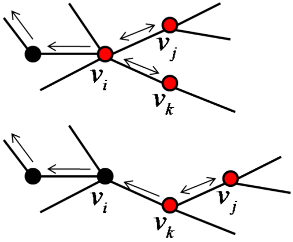


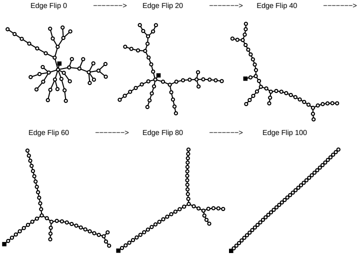
$$\mathcal{I}(v_5) = \mathcal{I}(v_6) = v_4$$
$$\mathcal{I}(v_2) = \{v_1, v_3\}$$

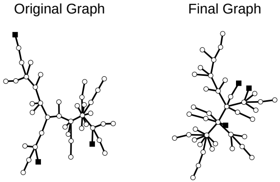
$\mathbf{E}(\tilde{x})$ Protocol (Increase)



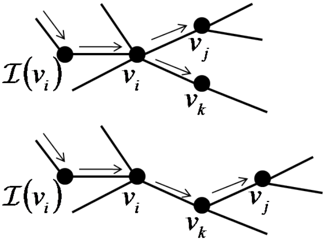
$\mathbf{Var}(x)$ Protocol (Decrease)



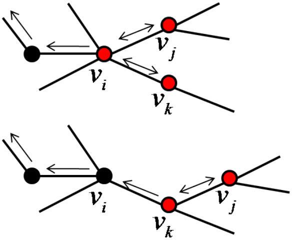


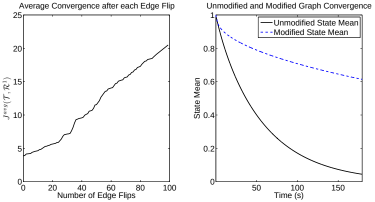


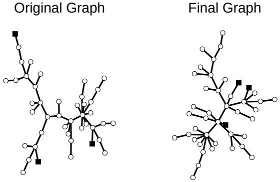
$\mathbf{E}(\tilde{x})$ Protocol (Increase)



$\mathbf{Var}(x)$ Protocol (Decrease)



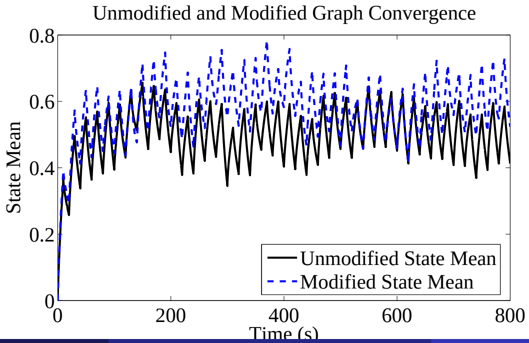




Time Synchronization Example - $J^{\text{avg}}(\mathcal{G}, \mathcal{R})$ Protocol

- 40 independent processes running on lab computers in an adaptable tree configuration on a standard TCP/IP network
- Running time consensus and the $J^{\text{avg}}(\mathcal{G}, \mathcal{R})$ increase/decrease protocol
- Switching $r = 3$ friendly/malicious agents delivering 1 sec and 0 sec respectively

Movie Link



Conclusion

- Provided links between the efficiency of semi-autonomous systems and the underlying network structure via metrics $J^{\mathrm{avg}}(\mathcal{G}, \mathcal{R})$ and $\mathbf{tr}P(\mathcal{G}, \mathcal{R})$.
- Proposed local protocols involving adjacent edge swaps that predictably alter these metrics.

Pending Questions:

- What about graphs that compromise between favorable $J^{\mathrm{avg}}(\mathcal{G}, \mathcal{R})$ and $\mathbf{tr}P(\mathcal{G}, \mathcal{R})$?
- Focused on constant mean noise, how about more arbitrary signals?
- How about simultaneous friendly and malicious attached agents?
- Local protocols over general graphs?