Semi-Autonomous Networks

Network Resilience and Adaptive Trees

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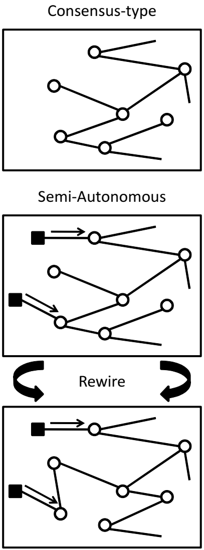
Motivation

- Network structure links to system properties within networked multi-agent system
- Particularly strong in consensus-type coordination algorithms
- Semi-autonomous systems are formed from consensus-type systems by introducing leaders/influencing agents

"How does network structure effect the performance of semi-autonomous systems?"

"Can you dynamically rewire the network structure to improve or degrade the performance of leaders/influencing agents?"



Outline

- Form the problem as a input-output dynamic system
- Introduce measures to quantify the system's performance to influencing agents
- Relate these measures to the network structure via an equivalent electrical network
- Form local edge swap protocols to alter these measures
- Use game theoretic techniques to quantify the protocol's success

Graphs and Consensus Model

- Graph structure: $\mathcal{G} = (V, E)$, where $V \in \mathbb{R}^n$ and $E \in \mathbb{R}^e$
- Matrix representation $L(\mathcal{G}) \in \mathbb{R}^{n \times n}$ where

$$[L(\mathcal{G})]_{ij} = \begin{cases} d_i & i = j \\ -1 & \{i,j\} \in E \\ 0 & otherwise \end{cases}$$

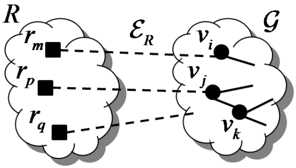
and d_i is the degree of node v_i.

Consensus Model

$$\dot{x}_i(t) = \sum_{\{i,j\} \in E} (x_j(t) - x_i(t)) \iff \dot{x}(t) = -L(\mathcal{G})x(t)$$

Semi-Autonomous Model

- Influencing node set $\mathcal{R} = (R, \mathcal{E}_R)$, $|\mathcal{E}_R| = r$
- Each agent in R is attached to exactly one agent in V
- Common mean, gaussian input



Dirichlet Matrix

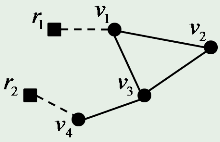
$$A(\mathcal{G}, \mathcal{R}) = -\left(L(\mathcal{G}) + B(\mathcal{R})B(\mathcal{R})^T\right)$$

Influence Model

$$\dot{x}(t) = A(\mathcal{G}, \mathcal{R})x(t) + B(\mathcal{R})u(t)$$

Example:

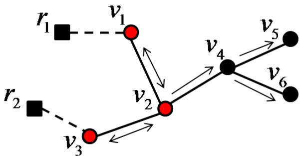
$$A(\mathcal{G}, \mathcal{R}) = \begin{bmatrix} -3 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -3 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix},$$



$$B(\mathcal{R}) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

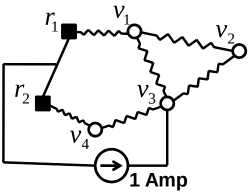
Nomenclature

- Attachment points: $\pi(\mathcal{E}_R)$ is the agents that the influence agents directly attach.
- Main Path agents: $\mathcal{M}$ is the set of agents that lie on any of the shortest paths between agents in $\mathcal{R}$.
- Main neighbors: $\Gamma(v_i)$ is the closest agents to v_i that is in $\mathcal{M}$.



$\pi(\mathcal{E}_R) = \{v_1, v_3\}$,
$\mathcal{M} = \{v_1, v_2, v_3\}$
$\Gamma(v_4) = \Gamma(v_5) = \Gamma(v_6) = v_2$

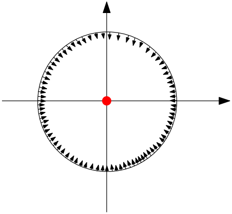
- Effective resistance: $E_{\text{eff}}(v_i)$ is the potential drop between v_i and $\mathcal{R}$, when a 1 Amp current source is connected across them.



Influence Measures

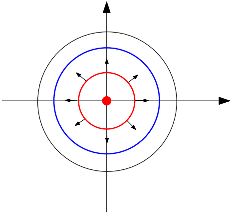
Mean tracking measure is the *average mean of the error* to steer the entire network to the origin over an infinite time horizon:

$$J_{\mu}(\mathcal{G},\mathcal{R}) = 2\mathbb{E}_{\|z(0)\|=1}\int_0^{\infty} z(t)^T z(t)dt.$$



Variance damping measure is the *average variance of the error* due to external agents injecting white noise as $t \to \infty$:

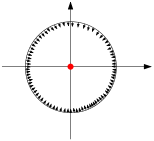
$$J_{\sigma}(\mathcal{G},\mathcal{R}) = \frac{2}{n}\lim_{t\to\infty}\mathbb{E}z(t)^T z(t).$$



Error signal $z(t) = x(t) - u_c\mathbf{1}$, where $u_c \in \mathbb{R}$ is the common mean of $u(t)$.

Mean Tracking Measure

$$J_\mu(\mathcal{G},\mathcal{R}) := 2\mathbb{E}_{\|z(0)\|=1}\int_0^\infty z(t)^T z(t)dt.$$



Equivalent expression

$$J_\mu(\mathcal{G},\mathcal{R}) = \frac{1}{n}\mathbf{tr}\left(-A(\mathcal{G},\mathcal{R})^{-1}\right),$$

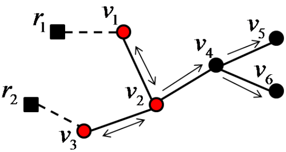
and another related to the graph structure through the effective resistance as

Effective Resistance Representation

$$J_\mu(\mathcal{G},\mathcal{R}) = \frac{1}{n}\sum_{i=1}^{n} E_{\text{eff}}(v_i).$$

A distance interpretation

$$J_\mu(\mathcal{T},\mathcal{R}) = \frac{1}{n}\Big(\sum_{v_i \in \mathcal{M}} E_{\text{eff}}(v_i) + \sum_{v_i \notin \mathcal{M}} \big[E_{\text{eff}}(\Gamma(v_i)) + d(v_i, \Gamma(v_i))\big]\Big).$$



A **single influence** agent attached to v_i

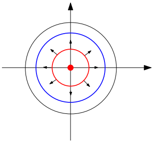
$$J_\mu(\mathcal{T},\mathcal{R}^i) = \frac{1}{n}\left(\sum_{j=1}^{n} d(v_i, v_j) + n\right) = \frac{n-1}{n} c(v_i, \mathcal{T}) + 1.$$

where $c(v_i, \mathcal{G})$ is the *closeness centrality* of v_i, i.e., the average distance between v_i and all other nodes in the graph.

- General Bounds for n-tree are $2 - \frac{1}{n} \leq J_\mu(\mathcal{T},\mathcal{R}^i) \leq \frac{1}{2}(n+1)$.

Variance Damping Measure

$$J_\sigma(\mathcal{G},\mathcal{R}) := \frac{2}{n}\lim_{t\to\infty}\mathbb{E}z(t)^T z(t).$$



Equivalent expression

$$J_\sigma(\mathcal{G},\mathcal{R}) = \frac{2}{n}\mathbf{tr}(P(\mathcal{G},\mathcal{R})),$$

where $P(\mathcal{G},\mathcal{R})$ is the controllability gramian

$$P(\mathcal{G},\mathcal{R}) := \int_0^\infty e^{A(\mathcal{G},\mathcal{R})\tau} B(\mathcal{G},\mathcal{R}) B(\mathcal{G},\mathcal{R})^T e^{A(\mathcal{G},\mathcal{R})^T\tau} dt,$$

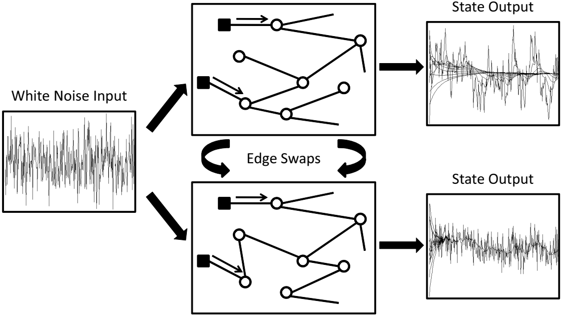
and another related to the graph structure through the effective resistance as

Effective Resistance Representation

$$J_\sigma(\mathcal{G},\mathcal{R}) = \frac{1}{n}\sum_{v_i\in\pi(\mathcal{E}_R)} E_{\text{eff}}(v_i).$$

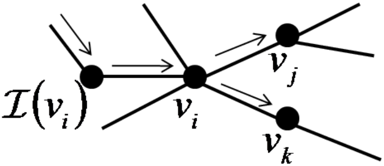
- A single external agent is attach to v_i for any $\mathcal{G}$ then, $J_\sigma\left(\mathcal{G},\mathcal{R}^i\right) = \frac{1}{n}$.

Network Design Problem



- Maintain connected graph
- Decentralized, parallel, asynchronous
- Local agent information of the graph structure

- $\mathcal{I}(v_i)$ is the neighbors of v_i closer to some influence agent than v_i.

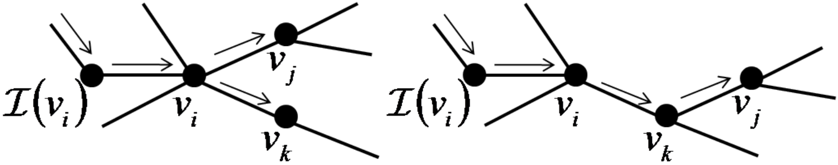


Balancing Measures

Relationship between measures

$$J_\mu(\mathcal{G},\mathcal{R}) = J_\sigma(\mathcal{G},\mathcal{R}) + \frac{1}{n}\sum_{v_i \notin \pi(\mathcal{E}_R)} E_{\text{eff}}(v_i).$$

- For security, large $J_\mu(\mathcal{G},\mathcal{R})$ (resistance to external control) and small $J_\sigma(\mathcal{G},\mathcal{R})$ (external noise damping) is favorable.
- Approach: For trees - increase $\sum_{v_i \notin \pi(\mathcal{E}_R)} E_{\text{eff}}(v_i)$ while keeping $J_\sigma(\mathcal{G},\mathcal{R})$ small.
- Compacts the main path (Protocol 3)
  Elongates the remaining graph (Protocol 1)

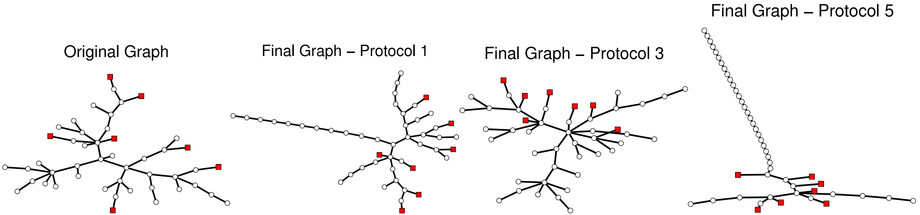


Distributed Protocol for Trees

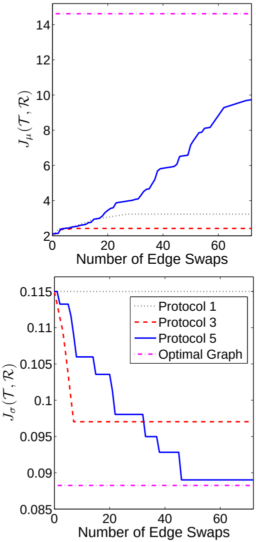
If $\exists v_j, v_k \in \mathcal{N}(v_i)$, $v_j \neq v_k$,

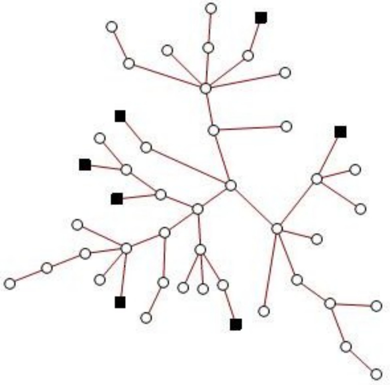
- Protocol 1: $\{(v_j, v_k \notin \mathcal{I}(v_i)\}$ - Increases $J_\mu(\mathcal{T}, \mathcal{R})$
- Protocol 3: $\{v_j, v_k \in \mathcal{I}(v_i)$ and $v_i \notin \pi(\mathcal{E}_R)\}$ - Decreases $J_\sigma(\mathcal{T}, \mathcal{R})$
- Protocol 5: either condition of Protocol 1 and 3 - Guarantees?

Then remove edge e_{ij} and add edge e_{jk}.



Simulation of Protocol 5





Price of Stability (PoS) and Anarchy (PoA)

Protocol 5 is a **finite signed potential game** $\implies$ at least one Nash equilibrium.

Definition

With the objective to minimize some function value then

$$\text{PoS} = \frac{\text{Best Equilibrium Value}}{\text{Optimal Value}} \text{ and PoA} = \frac{\text{Worst Equilibrium Value}}{\text{Optimal Value}}.$$

For protocol 5:

- With cost $1/J_\mu(\mathcal{T},\mathcal{R})$ the PoS$=1$ and PoA$\leq r$.
- With cost $J_\sigma(\mathcal{T},\mathcal{R})$ the PoS$=1$ and PoA$< \frac{11\sqrt{5}}{20} \approx 1.23$.

(Consequence: For $r=1$, protocol 5 will always reach the optimal value for $1/J_\mu(\mathcal{T},\mathcal{R})$.)

Conclusion

- Provided links between the efficiency of semi-autonomous systems and the underlying network structure via measures $J_{\mu}(\mathcal{G},\mathcal{R})$ and $J_{\sigma}(\mathcal{G},\mathcal{R})$.
- Proposed local protocols involving adjacent edge swaps that predictably alter these measures.

Pending Questions: How about...

- simultaneous friendly and malicious influence agents?
- generalized graphs?
- intelligent influencing agents?