Errata: Controllability and Observability of Network-of-Networks via Cartesian Products

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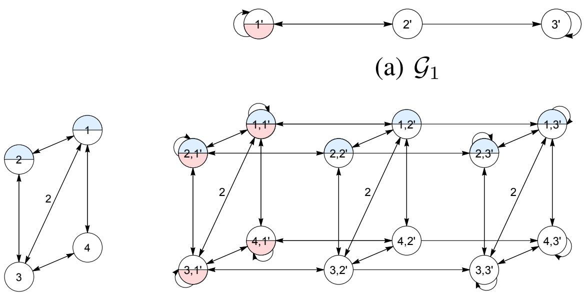
***Abstract*—Graph $\mathcal{G}_1$ was incorrectly represented in Figure 1 and Example IV.18. Many thanks to Yuqing Hao at Peking University for bringing this to our attention.**

II. Notation and Background



(a) $\mathcal{G}_1$

(b) $\mathcal{G}_2$

(c) $\mathcal{G} = \mathcal{G}_1 \square \mathcal{G}_2$

Fig. 1. Factor graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ and composite graph $\mathcal{G}_1 \square \mathcal{G}_2$. Edge weights of all graphs are 1 unless otherwise marked. The shading on the nodes pertains to Example IV-5, IV-15 and IV-18. (a) $\mathcal{G}_1$; (b) $\mathcal{G}_2$; (c) $\mathcal{G}_1 \square \mathcal{G}_2$.

IV. Controllability of Cartesian Product Networks

Example .1. *IV.18: Define the matrix representation*

$$[A(\mathcal{G})]_{ij} = \begin{cases} -w_{ji} & \text{for } i \neq j \\ -20\sum_{i=1}^{n} w_{ii} + 11\sum_{i\neq j} w_{ji} & \text{otherwise} \\ \quad -10\sum_{i\neq j} w_{ji}. & \end{cases}$$

In a more compact form $A(\mathcal{G}) = \mathscr{L}(\mathcal{G}) - 20\mathscr{D}_s(\mathcal{G}) + 10(\mathscr{D}_{in}(\mathcal{G}) - \mathscr{D}_{out}(\mathcal{G}))$. *Hence, as* $\mathbf{A}_{\oplus}$ *is closed under addition then* $A(\cdot) \in \mathbf{A}_{\oplus}$. *Consider the graphs* $\mathcal{G}_1$ *and* $\mathcal{G}_2$ *depicted in Fig. 1. Then* $A(\mathcal{G}_2) = \mathscr{L}(\mathcal{G}_2)$ *and so has eigenvalues* $0 = \mu_1 < \mu_2 \leq \mu_3 \leq \mu_4$ *and*

$$A(\mathcal{G}_1) = \begin{bmatrix} -19 & -1 & 0 \\ -1 & -9 & 0 \\ 0 & -1 & -9 \end{bmatrix},$$

which has all strictly negative eigenvalues. From (6) it follows that $\lambda_{min}(P(A(\mathcal{G})), B_1 \otimes I) = \lambda_{min}(P(A(\mathcal{G}_1), B_1))$. *Therefore, the optimal single node input set* S_1 *in terms of the smallest controllability Gramian eigenvalue for the graph* $\mathcal{G}_1$ *generates the optimal set* S *for the graph* $\mathcal{G}$ *of the form* $B_1(S_1)\square I_4$. *The smallest eigenvalue of* $P(A(\mathcal{G}_1), B_1)$ *from single input nodes* $1'$, $2'$ *and* $3'$ *are* 0.0218×10^{-5}, 0.3653×10^{-5}, *and* 0, *respectively. Thus the optimal set in terms of the smallest controllability Gramian eigenvalue is* $S_1 = \{2'\}$ *for* $\mathcal{G}_1$ *and* $S = \{\{1, 2'\}, \{2, 2'\}, \{3, 2'\}, \{4, 2'\}\}$ *for* $\mathcal{G}$.

A preliminary version of this paper appeared in the *Proceeding of the 51st IEEE Conference on Decision and Control* [1].

References

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