Errata: State Controllability, Output Controllability and Stabilizability of Networks: A Symmetry Perspective

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***Abstract*—Prof. Kanat Camlibel kindly pointed out with an illuminating couterexample that the PBH test for output controllability quoted in Proposition 1 was incorrectly referenced. This PBH style test provides a sufficient condition for output uncontrollability and output unstabilizability not a necessary condition. We have corrected Proposition 1 accordingly and Theorem 2 which follows from Proposition 1.**

4. Output Controllability and Stabilizability

Proposition 1. *Popov-Belevitch-Hautus (PBH) test [1]. For $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$. Consider one or more of the following conditions on $\lambda \in \mathbb{C}$ and $v \in \mathbb{C}^n$:*

(a) $v^T B = 0$,

(b) $\exists q \in \mathbb{R}^p$ such that $q^T C = v^T$ and

(c) Re$(\lambda) \geq 0$.

Then there exists a left eigenvalue-eigenvector pair (λ, v) of A such that

1. (a) $\iff$ (A, B) is uncontrollable.

*2. (a) and (b) $\implies$ (A, B, C) is output uncontrollable. **(Only sufficient!!!)***

3. (a) and (c) $\iff$ (A, B) is unstabilizable.

*4. (a), (b) and (c) $\implies$ (A, B, C) is output unstabilizable. **(Only sufficient!!!)***

Theorem 2. *For A diagonalizable and C full row rank, consider one or more of the following conditions on a $P \in \mathbb{R}^{n\times n}$:*

(a) $P \neq I$, $AP = PA$ and $PB = B$.

(b) $P = I - ZC$, for some $Z \in \mathbb{R}^{n\times p}$.

(c) $\frac{1}{2}(P + P^T) \preceq I$ and $PA + (PA)^T \preceq A + A^T$.

Then there exists a $P \in \mathbb{C}^{n\times n}$ such that

1. (a) $\iff$ (A, B) is uncontrollable.

*2. (a) and (b) $\implies$ (A, B, C) is output uncontrollable. **(Only sufficient!!!)***

3. (a) and (c) $\implies$ (A, B) is unstabilizable.

4. (a), (b) and (c) $\implies$ (A, B, C) is output unstabilizable.

*Further, for symmetric A, the statements 3. ~~**and 4.**~~ are necessary and sufficient.*

References

[1] T. Kailath, *Linear Systems*. Upper Saddle River: Prentice Hall, 1979.