2015 IEEE International Conference on Robotics and Automation (ICRA)
Washington State Convention Center
Seattle, Washington, May 26-30, 2015

Efficient Leader Selection for Translation and Scale of a Bearing-Compass Formation

Eric Schoof, Airlie Chapman, and Mehran Mesbahi

Abstract—**The paper considers the efficient selection of leader agents in a swarm running a distributed bearing-compass formation controller. The leaders apply external control which induces translation and scaling of the formation, providing manipulation methods useful to a human operator. The selection algorithm for maximizing translation and scale draws from modularity and submodularity theory. Consequently, the algorithms exhibit guaranteed optimal and suboptimal performance, respectively. For more restricted human-swarm interaction requiring pure translation and scale, a relaxed integer programming algorithm is described to reduce the combinatorial optimization problem to a computationally tractable semi-definite program. The leader selection strategies are supported through demonstration on a swarm testbed.**

Index Terms—**Distributed Control; Human-Swarm Interaction; Bearing-Only Control; Leader-Selection**

I. Introduction

The design of control algorithms to create, maintain and manipulate a formation of multi-agent networks is an important problem in robotics. Significant of late is the problem's formulation from a distributed point-of-view, where an individual robotic agent only uses local information for its operation [1], [2]. The emphasis of robot self-reliance over global communication dependence creates dynamics that are more robust to agent failure and scalable to larger numbers of agents.

There are many approaches to distributed formation control, characterized by the states an agent can observe of its neighbors. A common strategy employed in the literature is to generate an agent control law that relies on sensor information obtained about other nearby agents or landmarks. One approach requires knowledge of the relative position of neighboring agents, either in Cartesian or polar coordinates [2]. This method is simple to implement, but requires relative position sensors and a common global reference frame. Another common approach requires only knowledge of the relative distance of neighboring agents [2], [3]. This information can be obtained for example by the strength of the received wireless communication signals. The method used in this work requires knowledge of the relative bearing of neighboring agents for example from a monocular camera [4]. A mix of range and bearing information can also be used, for example when a subset of agents has the full relative position of a subset of neighbors [5], or when the distance between agents can be estimated using bearing measurements [6].

There are many works in the literature that use a bearing-only approach to formation control [7], [8], [9]. We complemented these works by designing a control law that also relies on a single additional sensor, a compass, to localize the rotation in the global reference frame [10], which was also recently studied by Zhao and Zelazo [11]. The proposed bearing-compass control law has the attraction that the centroid location and scale remain invariant under the dynamics. A feature of interest to a human operator is the manipulation of the formation translation and scale. If external control is added to a subset of agents, referred to as leader agents, the centroid and scale can be arbitrarily shifted. A focus of this paper is the efficient selection of leader agents for achieving these tasks.

The leader selection problem for robotic swarms fits in a larger class of robot task assignment problems [12]. These problems are often posed as optimization problems with a set of leaders having an associated selection cost, such as swarm manipulability [13], network coherence [14] and $\mathcal{H}_2$ norm input amplification [15], [16]. The problems, due to their combinatorial nature, are often NP-hard requiring a relaxed or approximate solution.

We formulate the leader selection problem, with a range of translation and scaling cost functions, as a combinatorial optimization problem. For translation and scale maximization the problem is associated with the maximization of a monotone increasing submodular function [17]. Mirroring similar approaches to solving leader selection problems over submodular functions [18], [19], a greedy leader selection algorithm is applied which provides guaranteed sub-optimal performance [20]. Further we formulate the leader selection problem over a *pure* translation cost function, meaning maximizing translation without formation scaling, as a mixed-integer quadratic program. An integer relaxation of the problem yields a sub-optimal solution in polynomial time. The *pure* scaling leader selection problem is similarly examined. To demonstrate the algorithms feasibility, the control law and leader selection process is implemented on a unicycle swarm.

The structure of the paper is as follows. In §II, we present notation and definitions used throughout this paper and our previously introduced bearing-compass model [10]. In §III we propose the translation and scale leader problem and provide optimal and sub-optimal leader selection algorithms. The

This research of the authors was supported by AFOSR grant FA9550-12-1-0203. The authors are with the Department of Aeronautics and Astronautics, University of Washington, WA 98105. Emails: {eschoof, airliec, mesbahi}@uw.edu.

978-1-4799-6923-4/15/$31.00 ©2015 IEEE

implementation of the selection process on a swarm testbed is examined in §IV. Finally, we present some concluding remarks in §V.

II. Background

In this section, a brief background is provided on the notation, dynamics and definitions that are used in this paper.

The 2-norm of a vector v is denoted $\|v\|$. The normalized vector of v is denoted $\hat{v} = \frac{v}{\|v\|}$. Given vectors $v, w \in \mathbb{R}^2$, the perpendicular of $v = [\ v_1 \quad v_2\ ]^T$ is $v^\perp = [\ v_2 \quad -v_1\ ]^T$.

An undirected graph $\mathcal{G} = (V, E)$ consists of a node set $i \in V$ with cardinality $|V| = n$ and an edge set $(i, j) \in E$ with cardinality $|E| = m$. We define the set of neighbors of agent i as $j \in \mathcal{N}(i)$ if $(i, j) \in E$, where $\mathcal{N}(i)$ is called the *neighborhood* of i. Given two nodes $i, j \in V$, dist (i, j) is the shortest path between i and j. Similarly, given a set of nodes $S \subseteq V$, dist (i, S) is the shortest path between i and any of the nodes in S.

In this paper, we use the term "node" to describe a particular element of $V(\mathcal{G})$. There is also a more loosely termed "agent" which refers to something (usually a vehicle) that can communicate or act *through* the edges of $E(\mathcal{G})$, and can have its own state. We define the agent positions in 2-D space at time t by the vector $r(t) = [\ r_1^T(t) \quad r_2^T(t) \quad \ldots \quad r_n^T(t)\ ]^T$, where $r_i(t) = [\ x_i(t) \quad y_i(t)\ ]^T$ and agent i is at position $(x_i(t), y_i(t))$. The vector from agent i to agent j is $r_{ij} = r_j - r_i$. The pair $(r, \mathcal{G})$ represents a set of agents that can communicate or observe each other over the edge set in $\mathcal{G}$. A desired formation shape, distinct up to scale and translation, is characterized using $2m$ bearing constraints of the form $\hat{f}_{ij} = [\ \cos(\phi_{ij}) \quad \sin(\phi_{ij})\ ]^T$, where ϕ_{ij} is the desired bearing of agent j from the perspective of agent i. The desired formation is compactly described by $\Theta(\mathcal{G}) := \{\hat{f}_{ij}\}$, for a fixed graph $\mathcal{G}$.

A. Bearing-Compass Particle Dynamics

In this paper, we are extending the analysis of our previously introduced bearing-compass particle dynamics [10]

$$\dot{r}_i = u_i(\Theta) + \tilde{u}_i \tag{1a}$$

$$u_i(\Theta) = \sum_{j \in \mathcal{N}(i)} (\hat{r}_{ij}^T \hat{f}_{ij}^\perp) \hat{r}_{ij}^\perp. \tag{1b}$$

where $\hat{f}_{ij} \in \Theta$ and $\tilde{u}_i \in \mathbb{R}^2$ is an external additive control input on agent i. The notation used in (1) is illustrated in Figure 1.

The dynamics are referred to as bearing-compass as they can be implemented with only a compass on each agent and relative bearing information to neighboring agents. The latter, can be achieved via an on-board low-fidelity camera where distance measurements can be unreliable compared to bearing measurements. Agents with $\tilde{u}_i \neq 0$ are referred to as *leader* agents, as through the networked dynamics, the non-leader agents follow the leaders.

Our previous results pertaining to dynamics (1) show the desired formation shape is almost exponentially stable [10].

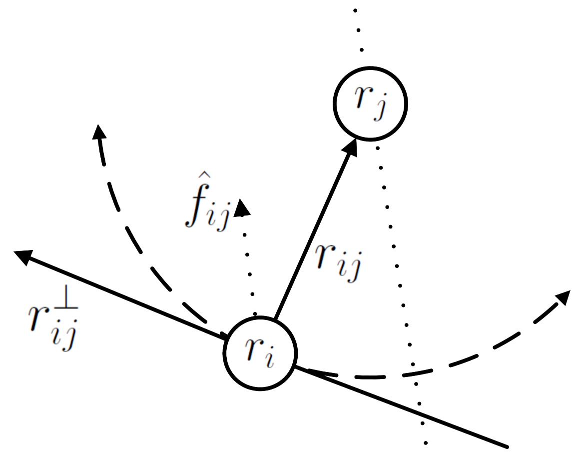

Figure 1: Model vector definitions.

Another feature of the dynamics is the invariance of the centroid location $C(r) = \frac{1}{n}\sum_{i \in V} r_i$ and scale $S(r) = \sum_{i \in V} \|r_i - C(r)\|^2$ under the unforced dynamics. When there are leader agents present in the network the centroid and scale can be manipulated for the now *forced* dynamics. The relationship between the change of centroid and scale can be characterized in terms of the leaders' external control and summarized in the following Proposition.

Proposition 1. *[10] Under the forced dynamics (1), the rate of change of the centroid and scale satisfy*

$$\frac{\partial C(r)}{\partial t} = \frac{1}{n} \sum_{i \in V} \tilde{u}_i$$

$$\frac{\partial S(r)}{\partial t} = 2 \sum_{i \in V} (r_i - C(r))^T \tilde{u}_i.$$

The objective of this paper is to apply Proposition 1 to provide leaders with a method of translating and scaling the formation.

B. Modular and Submodular Functions

Let $S \subseteq V$ be the set of leader agents participating in a particular maneuver. The effectiveness of the set S to complete the task is denoted as $f(S)$ where f is the set function $f : 2^n \to \mathbb{R}$. Three classes of set functions of interest in this paper are modular, monotone increasing and submodular functions with the following definitions [20].

Definition 2. A set function $f : 2^n \to \mathbb{R}$ is *modular* if and only if for any subset $S \subseteq V$ it can be expressed as

$$f(S) = w(\emptyset) + \sum_{s \in S} w(s),$$

for some weight function $w : V \to \mathbb{R}$.

Definition 3. A set function $f : 2^n \to \mathbb{R}$ is monotone increasing if, for all subsets $A \subseteq B \subseteq V$ it holds that $f(A) \leq f(B)$.

Definition 4. A set function $f : 2^n \to \mathbb{R}$ is *submodular* if for all subsets $A \subseteq B \subseteq V$ and $s \notin B$ then $f(A \bigcup \{s\}) - f(A) \geq f(B \bigcup \{s\}) - f(B)$.

The intuition behind Definition 4 is that adding an element to a smaller set gives a greater gain than adding to a larger set. Submodular functions bare similarities to convex and concave functions [17]. This connection can be used to provide guarantees on greedy algorithms optimizing over submodular functions [20].

III. Translation and Scale Manipulation

One of the attributes previously explored for dynamics (1) is the application of external control on all or a subset of the agents referred to as *leader* agents to manipulate the formation. Motivated by *intuitive* human-swarm formation interactions, we focused on control strategies that can translate and scale the formation. As specific controls are required to be communicated to each agent to perform these strategies, communication costs can restrict the number of agents able to participate in a maneuver. When constrained to a finite control, some agents can more effectively instigate these maneuvers. Further, translation without scaling, referred to as *pure* translation, can be achieved by applying carefully selected external leader control. Similarly, translation can be maintained while varying scale when the external leader control values are restricted. These observations indicate that an intelligent selection of control agents and control signals is required when interacting with the swarm.

This section discusses solutions and approximate solutions for the leader set S selection problem for effective formation translation and scaling.

A. Translation and Scale Maximization

As mentioned in Proposition 1, the change in the centroid is

$$\frac{\partial C}{\partial t} = \frac{1}{n}\sum_{i\in S}\tilde{u}_i,$$

which provides a direct method for leader agents to translate the formation. Consider a translation of the formation in the direction of the unit vector $h \in \mathbb{R}^2$. If each agent can apply a cumulative unforced and forced control of magnitude at most $u_{\max}$ then, as the unforced dynamics of agent i is at most $\mathcal{N}(i)$ [10], it follows that $\|\tilde{u}_i\| \le u_{\max} - |\mathcal{N}(i)|$. Here, we assume there is sufficient control to apply the unforced dynamics, i.e., $u_{\max} \ge \mathcal{N}(i)$. The set function describing the most suitable translation leader set S is

$$\begin{aligned} f_t(S) &= \max_{\|\tilde{u}_i\|\le u_{\max}-|\mathcal{N}(i)|} h^T\left(\frac{1}{n}\sum_{i\in S}\tilde{u}_i\right) \\ &= \frac{1}{n}\sum_{i\in S}(u_{\max}-|\mathcal{N}(i)|), \end{aligned} \tag{2}$$

corresponding to $\tilde{u}_i = h\,(u_{\max}-\mathcal{N}(i))$ for each agent i. For this case, every agent applies their maximum control in the direction of h.

A similar setup occurs for maximizing the change in scale of the formation, defined as

$$\frac{\partial S}{\partial t} = 2\sum_{i\in V} q_i^T\tilde{u}_i,$$

where $q_i = r_i - C(r)$. Under the cumulative control constraint $u_{\max}$ then

$$\begin{aligned} f_s(S) &= \max_{\|\tilde{u}_i\|\le u_{\max}-|\mathcal{N}(i)|} 2\sum_{i\in S} q_i^T\tilde{u}_i \\ &= 2\sum_{i\in S}(u_{\max}-|\mathcal{N}(i)|)\,\|q_i\|, \end{aligned} \tag{3}$$

corresponding to $\tilde{u}_i = (u_{\max}-|\mathcal{N}(i)|)\,\hat{q}_i$ for each agent i. The control strategy corresponds to every leader applying their maximum control outward from the direction of the centroid $\hat{q}_i$. The minimization of the change in scale of the formation can be similarly formulated. In fact if $\tilde{u}_i$ is the solution of the maximization problem then $-\tilde{u}_i$ is the solution to the minimization of change in scale.

Set functions $f_t(S)$ and $f_s(S)$ are in fact both modular functions from Definition 2. Maximization over modular functions with cardinality constraints, e.g. $|S| \le k$, is easy, requiring evaluation of $w(s)$ over all agents s and then selecting the k largest. Consequently, for the centroid set function $f_t(S)$ the best agent set is composed of agents with the k smallest degrees. Similarly, the best agent set for scale manipulation based on $f_s(S)$ is composed of agents that are far from the center with small degrees. For example, if all agents have equal degree than the optimal set will be composed of the k agents furthest from the center.

B. Information Propagation Maximization

The detriment of the optimal leader sets of the previous section is that leader agents often have a low degree which tends to imply a small distance centrality, i.e., $\sum_{i=1}^{n}\mathrm{dist}(i,S)$ is large. Consequently, the translation or scale information presented by agents in S can be slow to propagate through the dynamics over the graph. One approach to mitigate this effect is to supplement the set functions $f_t(S)$ and $f_s(S)$ with a centrality-promoting measure $f_{\mathcal{G}}(S)$ by optimizing over both namely

$$f(S) = f_x(S) + f_{\mathcal{G}}(S), \tag{4}$$

where $f_x(S)$ is either $f_t(S)$ or $f_x(S)$. Here,

$$f_{\mathcal{G}}(S) = -c\sum_{j=1}^{n}\mathrm{dist}(j,S) \tag{5}$$

for some $c > 0$ which penalizes the set S for large distance centrality.

The set function $f_{\mathcal{G}}(S)$ falls into a class of set functions known as monotone increasing, submodular functions. The submodularity of $f_{\mathcal{G}}(S)$ follows from the relation

$$\begin{aligned} & f_{\mathcal{G}}(B\bigcup\{s\}) - f_{\mathcal{G}}(B) \\ &= c\sum_{j=1}^{n}\max\{0,\mathrm{dist}(j,B)-\mathrm{dist}(j,s)\} \\ &\le c\sum_{j=1}^{n}\max\{0,\mathrm{dist}(j,A)-\mathrm{dist}(j,s)\} \\ &= f_{\mathcal{G}}(A\bigcup\{s\}) - f_{\mathcal{G}}(A), \end{aligned}$$

and the monotonicity of $f_{\mathcal{G}}(S)$ follows from the observation that adding a vertex to a set can never increase the sets distance to agents in the graph.

Applying the property that the sum of a submodular function and a modular function is submodular [17], then (4) is also a monotone increasing, submodular function. Now, the maximization of such a monotone increasing, submodular function for $|S| \leq k$ is generally NP-hard. In fact, a special case of maximizing $f_{\mathcal{G}}(S)$ is the vertex cover problem shown to be NP-complete [21]. A polynomial time alternative is to apply a greedy algorithm which is provably close to the optimal [20]. The greedy algorithm involves starting with S empty and adding the most lucrative nodes not already in S, i.e. $s \notin S$ with largest $f(s)$. The greedy solution S compared to the optimal S^* has the guaranteed bound [20]

$$\frac{f(S^*) - f(S)}{f(S^*) - f(\emptyset)} \leq \frac{1}{e} \approx 0.37.$$

In fact, this is the best performing polynomial time algorithm over general monotone increasing submodular functions [20].

An alternative candidate for $f_{\mathcal{G}}(S)$ is the cut size corresponding to the partition S and $V - S$. This is the number of edges linking the input node set S to the rest of the swarm. It presents an different measure of connectedness and therefore information propagation. A special case of maximizing $f_{\mathcal{G}}(S)$ is the maximum cut problem, one of Karp's 21 NP-complete problems [21]. The cut size is also a monotone increasing, submodular function and so a greedy heuristic can be applied with guaranteed suboptimal performance.

C. Pure Translation and Scale

A consequence of the control strategies corresponding to the set functions (2) and (3) is that the formation can be indirectly scaled when the centroid is translated and similarly centroid translation can inadvertently occur under scaling. This is particularly detrimental in the case of scale variation, as over time, $q_i(r)$ varies with centroid location, so unless the forced additive control is applied instantaneously the optimal S under the set function (3) may not in fact be the optimal scaling set.

An alternative is to select $\tilde{u}_i$ for all $i \in S$ such that only a pure translation or scale occurs. Formally, the control input $\tilde{u}_i$ is designed such that $\frac{\partial S(r)}{\partial t} = 0$ and $\frac{\partial C(r)}{\partial t} = 0$ for scale and centroid invariance, respectively.

For optimal translation leader selection under this restriction, an alternative set function can be considered which optimizes for pure translation in the desired direction h, while exhibiting no translation in other directions or scaling. The set function is defined, via an optimization problem over all $\tilde{u}_i$ such that $i \in S$, as

$$\begin{aligned} f_{t'}(S) \quad &= \quad \max h^T \sum_{i \in S} \tilde{u}_i & (6) \\ \text{s.t.} \quad & \sum_{i \in S} q_i^T \tilde{u}_i = 0 & (7) \\ & (h^{\perp})^T \sum_{i \in S} \tilde{u}_i = 0 & (8) \\ & \|\tilde{u}_i\| \leq u_{\max} - \mathcal{N}(i), \text{ for all } i \in S. \end{aligned}$$

The equality constraint (7) mandates that the scale remains invariant while (8) prohibits translations not aligned with h. Unlike set function (2), $f_{t'}(S)$ does not have a closed form solution. Given the set S, $f_{t'}(S)$ can be solved efficiently as a semidefinite optimization problem, or more specifically as a linear cost problem with linear and quadratic constraints. Unfortunately, $f_{t'}(S)$ is not submodular and so the techniques of the previous section can not be applied to form a $1 - 1/e$ approximate leader selection algorithm with cardinality constraints on S.

The problem of leader selection under the cost $f_{t'}(S)$ can be posed as an mixed-integer quadratic programming (MIQP) problem, optimizing over $\tilde{u}_1, \ldots, \tilde{u}_n$ and binary variables $z_1, \ldots, z_n$, as

$$\begin{aligned} J_{t'} \quad &= \quad \max h^T \sum_{i=1}^{n} \tilde{u}_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} q_i^T \tilde{u}_i = 0 \\ & (h^{\perp})^T \sum_{i=1}^{n} \tilde{u}_i = 0 \\ & \|\tilde{u}_i\| \leq z_i \left(u_{\max} - \mathcal{N}(i)\right), \text{ for all } i \in V & (9) \\ & \sum_{i=1}^{n} z_i \leq k & (10) \\ & z_i \in \{0, 1\}, \text{ for all } i \in V. & (11) \end{aligned}$$

The binary variable z_i, through inequality (9), captures whether the agent i is active, while the inequality (10) enforces the cardinality constraint, $|S| \leq k$. This problem is difficult to solve exactly. In fact for $u_{\max} \to \infty$, the problem is a 0-1 integer program which is NP-complete [21]. One approach is to find an approximate solution by performing an integer relaxation of constraint (11) to $z_i \in [0, 1]$. The optimal $J_{t'}$ will be generated by some contribution of all agents denoted through the magnitude of z_i. Selecting the agents which correspond to the largest k elements of z_i form an approximate agent set S. Defining the optimal agent selection as $S^* = \text{argmax}_{|S| \leq k} f_{t'}(S)$ then

$$0 \leq f_{t'}(S) \leq f_{t'}(S^*) \leq J_{t'}.$$

Hence, we have the relation

$$\frac{f_{t'}(S^*) - f_{t'}(S)}{f_{t'}(S^*)} \leq \frac{J_{t'} - f_{t'}(S)}{f_{t'}(S)}, \tag{12}$$

thus for small $J_{t'} - f_{t'}(S)$, S serves as a good approximation of S^*. The additive control $\tilde{u}_i$ for $i \in S$, for this centroid maneuver corresponds to the optimal solution of (6).

A similar approach can be adopted for the selection of leader controls for pure scaling with centroid invariance using an alternative set function

$$\begin{aligned} f_{s'}(S) &= \max \sum_{i\in S} q_i^T \tilde{u}_i \quad (13)\\ \text{s.t.} \quad & \sum_{i\in S} \tilde{u}_i = 0 \\ & \|\tilde{u}_i\| \le u_{\max} - \mathcal{N}(i), \text{ for all } i \in S. \end{aligned}$$

The accompanying MIQP problem, which for $u_{\max} \to \infty$ is an NP-complete 0-1 integer program, is

$$\begin{aligned} J_{s'} &= \max \sum_{i=1}^{n} q_i^T \tilde{u}_i \quad (14)\\ \text{s.t.} \quad & \sum_{i=1}^{n} \tilde{u}_i = 0 \\ & \|\tilde{u}_i\| \le z_i \left(u_{\max} - \mathcal{N}(i)\right), \text{ for all } i \in V \\ & \sum_{i=1}^{n} z_i \le k \\ & z_i \in \{0,1\}, \text{ for all } i \in V. \end{aligned}$$

The integer relaxation approach produces a similar inequality for the suboptimal scale maneuver

$$\frac{f_{s'}(S^*) - f_{s'}(S)}{f_{s'}(S^*)} \le \frac{J_{s'} - f_{s'}(S)}{f_{s'}(S)}, \quad (15)$$

where $S^* = \text{argmax}_{|S|\le k} f_{s'}(S)$.

IV. Examples

The leader selection algorithms were exercised on a unicycle testbed with the objective of demonstrating the scaling leader selection process. The testbed is composed of differential drive robots with positional information supplied by a Vicon motion capture system. A unicycle particle tracking dynamics coupled with dynamics (1) was implemented on the testbed, with bearing measurement $\hat{r}_{ij}$ supplied to agent i via the Vicon system. Analysis of these coupled dynamics will appear in future work.

The dynamics was applied to three vehicles with initial vehicle positions forming a right triangle and graph $\mathcal{G}$ a complete graph. The desired formation shape, defined through the set $\Theta(\mathcal{G})$, corresponds to a right-triangle in a different orientation. The resultant trajectory is depicted in Figure 2. The final formation shape is correctly acquired and demonstrates good agreement between the particle tracking dynamics and compass-bearing dynamics (1).

To examine the scaling leader selection process, the dynamics were applied to a 9 vehicle swarm. The initial formation shape and scale, and graph $\mathcal{G}$ are provided in Figure 3a. With the objective of maximizing the scale of the formation, a greedy leader selection algorithm was implemented over the modular cost function (3), selecting the optimal leader set with cardinality three. The consequent formation scaling is depicted in Figure 3a with the scale motion and centroid motion illustrated in Figure 3b and 3c, respectively. As expected, the selected leaders are those furthest from the centroid with low degree. The corresponding control vectors

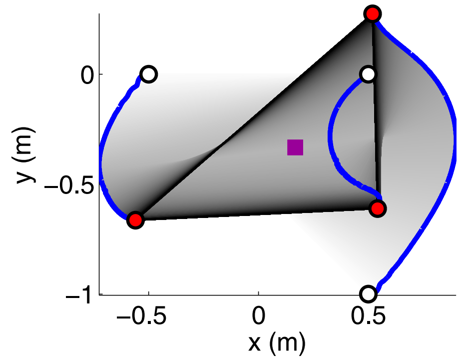

Figure 2: Initial and final positions are marked with empty and filled circles, respectively, and the centroid by a solid square. The trajectory of the agents is denoted with a solid line between empty and filled circles. The graph topology is indicated by lines between filled circles.

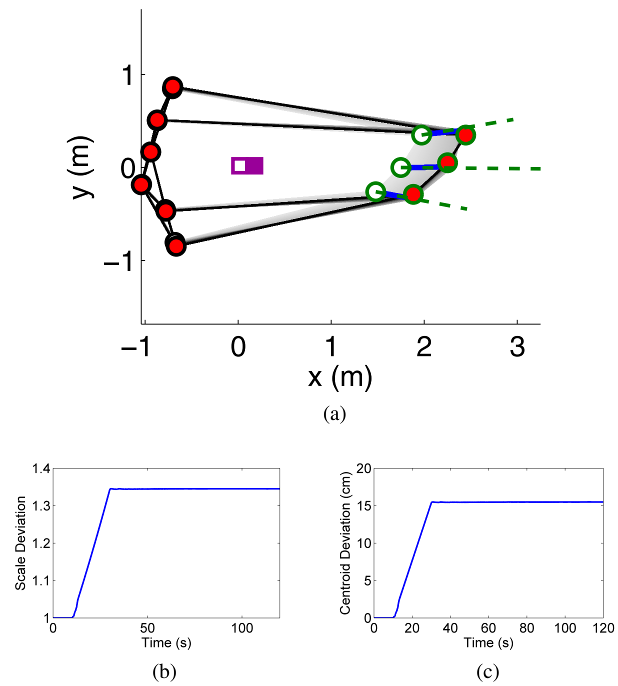

Figure 3: (a) A 9 agent formation under scale with leader selection dictated by cost function (3). (b) and (c) The resultant scale $S(r)$ and centroid $C(r)$ deviations over time. The initial position, final position, trajectory, graph, and centroid are denoted as in Figure 2. The control vector $\tilde{u}_i$ for each leader agent i is indicated with a dashed line from node i.

$\tilde{u}_i$, supplied for $t \in [10, 30]$, move the leader agents away from the centroid and in doing so scale the swarm. We observe an eventual 35% scaling of formation. There is also an inadvertent shift of the centroid by 15 cm.

An alternate cost function (13) was examined so as to avoid the unwanted shift in centroid observed in Figure 3c. The proposed integer relaxed optimization problem (14) explored in §III was solved to generate a sub-optimal leader set with effectiveness of these selections was illustrated on a unicycle swarm testbed. The subsequent testbed implementation demonstrated the merits of the proposed selection methods providing good human operator interfaces to the swarm. Plans for future work involve the extension of the centralized leader selection process to a distributed one.

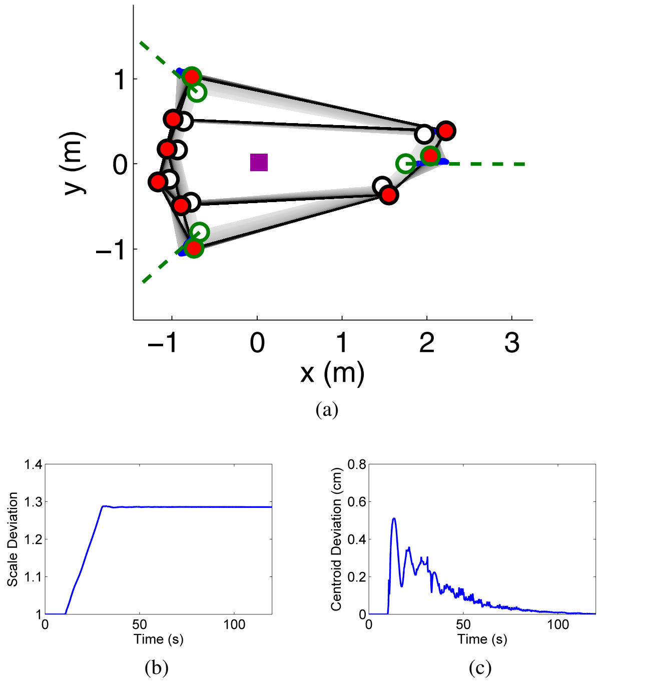

Figure 4: (a) A 9 agent formation under scale with leader selection dictated by cost function (13). (b) and (c) The resultant scale $S(r)$ and centroid $C(r)$ deviations over time. The initial position, final position, trajectory, graph, and centroid are denoted as in Figure 2. The control vector $\tilde{u}_i$ for each leader agent i is indicated with a dashed line from node i.

cardinality three. The bound (15) is

$$\frac{J_{s'} - f_{s'}(S)}{f_{s'}(S)} \leq 0.31,$$

indicating the solution is within 31% of the optimal. Examining all possible leader sets, this solution is in fact optimal. The subsequent trajectory of the same 9 agent scenario in Figure 3 is depicted in Figure 4a. Unlike in Figure 3, the selected leaders are not those that are furthest from the centroid, but balanced around it. The corresponding leader controls, supplied for $t \in [10, 30]$, also no longer point directly away from the centroid as they must enforce the centroid invariant constraints. The scale deviation, depicted in Figure 4b, shows a scaling of 30%. Figure 4c illustrates the more favorable centroid deviation with only a 0.5 cm shift from the initial location opposed in 15 cm for cost function (3).

V. Conclusion

In this work we examined the leader selection problem for a compass-bearing control law. The criteria for selection was based on the leaders' effectiveness to manipulate the formation's centroid location and scale. The formulation of the problem as the maximization of a monotone increasing submodular function and a relaxed integer program provides methods to efficiently select good leader sets. The

References

[1] R. Olfati-Saber and R. M. Murray, "Distributed cooperative control of multiple vehicle formations using structural potential functions," in *IFAC World Congress*, 2002.

[2] M. Mesbahi and M. Egerstedt, *Graph Theoretic Methods in Multiagent Networks*. Princeton: Princeton University Press, 2010.

[3] M. Egerstedt and X. Hu, "Formation contrained multi-agent control," *IEEE Transactions on Robotics and Automation*, vol. 17, no. 6, pp. 3961–3966, 2001.

[4] A. Huster, E. Frew, and S. Rock, "Relative position estimation for AUVs by fusing bearing and inertial rate sensor measurements," in *Oceans '02 MTS/IEEE*, 2002, pp. 1863–1870.

[5] A. N. Bishop, T. H. Summers, and B. D. O. Anderson, "Control of triangle formations with a mix of angle and distance constraints," in *IEEE International Conference on Control Applications*, 2012, pp. 825–830.

[6] G. L. Mariottini, F. Morbidi, S. Member, N. V. Valk, and G. Pappas, "Vision-Based Localization for Leader-Follower Formation Control," *IEEE Transactions on Robotics*, vol. 25, no. 6, pp. 1431–1438, 2009.

[7] A. N. Bishop, I. Shames, and B. D. O. Anderson, "Stabilization of rigid formations with direction-only constraints," in *IEEE Conference on Decision and Control and European Control Conference*, 2011, pp. 746–752.

[8] A. Franchi and P. R. Giordano, "Decentralized control of parallel rigid formations with direction constraints and bearing measurements," in *IEEE Conference on Decision and Control*, 2012, pp. 5310–5317.

[9] N. Moshtagh, N. Michael, A. Jadbabaie, and K. Daniilidis, "Vision-Based, Distributed Control Laws for Motion Coordination of Nonholonomic Robots," *IEEE Transactions on Robotics*, vol. 25, no. 4, pp. 851–860, 2009.

[10] E. Schoof, A. Chapman, and M. Mesbahi, "Bearing-compass formation control: A human-swarm interaction perspective," in *2014 American Control Conference*, 2014, pp. 3881–3886.

[11] S. Zhao and D. Zelazo, "Bearing rigidity and almost global bearing-Only formation stabilization," *arXiv preprint arXiv: arXiv:1408.6552*, pp. 1–19, 2014.

[12] N. Michael, M. M. Zavlanos, V. Kumar, and G. J. Pappas, "Distributed Multi-Robot Task Assignment and Formation Control," *2008 IEEE International Conference on Robotics and Automation*, vol. 1, pp. 128–133, 2008.

[13] H. Kawashima and M. Egerstedt, "Leader Selection via the Manipulability of Leader-Follower Networks," *2012 American Control Conference*, pp. 6053–6058, 2012.

[14] S. Patterson and B. Bamieh, "Leader selection for optimal network coherence," *Proc. 49th IEEE Conference on Decision and Control*, pp. 2692–2697, 2010.

[15] A. Chapman and M. Mesbahi, "Semi-autonomous consensus: network measures and adaptive trees," *IEEE Transactions on Automatic Control*, vol. 58, no. 1, pp. 19–31, 2013.

[16] F. Lin, M. Fardad, and M. R. Jovanović, "Algorithms for leader selection in stochastically forced consensus networks," *IEEE Transactions on Automatic Control*, vol. 59, no. 7, pp. 1789–1802, 2014.

[17] L. Lovász, "Submodular functions and convexity," in *Mathematical Programming The State of the Art*. Berlin: Springer, 1983, pp. 235–257.

[18] A. Clark and R. Poovendran, "A submodular optimization framework for leader selection in linear multi-agent systems," in *Proc. 50th IEEE Conference on Decision and Control*, 2011, pp. 3614–3621.

[19] F. L. Cortesi, T. H. Summers, and J. Lygeros, "Submodularity of Energy Related Controllability Metrics," in *Proc. 54th IEEE Conference on Decision and Control*, 2014, pp. 2883 – 2888.

[20] G. Nemhauser, L. Wolsey, and M. Fisher, "An analysis of approximations for maximizing submodular set functions I," *Mathematical Programming*, vol. 14, pp. 265–294, 1978.

[21] M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*, 1979.