Problem 1. Consider a time invariant, second order system:

\[ m\ddot{y}(t) + c\dot{y}(t) + ky(t) = 0. \]

Then, the natural frequency \( \omega_n \) of the system is obtained by \( \omega_n = \sqrt{k/m} \).

(a) Given the following characteristics for \textit{Seasat} spacecraft, estimate the natural frequencies of pitch motion due to gravity gradient forces (by Earth).

\begin{align*}
I_1 &= I_2 = 25,100kg \cdot m^2, \\
I_3 &= 3000kg \cdot m^2 \\
\Omega &= 0.00105 \text{rad/s}.
\end{align*}

(b) Simulate the coupled nonlinear response of the spacecraft to an initial yaw rate disturbance of \( 10^{-5} \) rad.

Problem 2. A dumbbell-shaped spacecraft (two masses \( m \) connected by a weightless bar of length \( L \)) as shown in Fig. 1 is subjected to the influence of an inverse square gravity field of the Earth. The circular orbit of the center of mass has a semi-major axis of 7000km. Express all vectors in the orbit frame \( O : \{\hat{i}_r, \hat{i}_\theta, \hat{i}_h\} \).

(a) Derive the gravity gradient torque for this simple two-dimensional spacecraft model about the craft center of mass. Have \( \theta \) be the angle of the craft with respect to nadir.

(b) With \( m = 100kg \) and \( L = 10m \), plot the gravity gradient torque magnitude for \(-\pi/2 < \theta < \pi/2\).

(c) Assume an atmospheric torque of \( \tau = 2 \times 10^{-3}Nm \) is acting on the body. At what torque equilibrium angle (TEA) \( \theta_{TEA} \) will the gravity gradient torque cancel the atmospheric torque?

![Figure 1: Dumbbell spacecraft illustration.](image)

Problem 3. Given a homogeneous cylinder of radius \( R \) and height \( h \), what is the smallest possible cylinder height such that this object would still be gravity gradient stabilized in attitude (assume height aligned with \( \hat{\theta} \))?
Problem 4. From Eq. (4.187), (a) verify the gravity gradient stability conditions in Eqs. (4.188-4.190). (b) Find an example of inertia matrix falling in Region II in Fig 4.19.

Problem 5. Consider the spacecraft with \( I_1 = 400 \text{kgm}^2 \), \( I_2 = 300 \text{kgm}^2 \), and \( I_3 = 200 \text{kgm}^2 \) on a circular orbit of radius \( r = 7000 \text{km} \). If the attitude of the spacecraft body frame \( B : \{\hat{b}_1, \hat{b}_2, \hat{b}_3\} \) differs from the orbit frame \( O : \{\hat{o}_1, \hat{o}_2, \hat{o}_3\} \) through a yaw of 30 deg, pitch of -10 deg, and roll of 20 deg (standard 3-2-1 Euler angles), what is the gravity gradient torque vector expressed in body frame components? Answer in units of Newton meters.

Problem 6. Consider the cylindrical spacecraft illustrated in Fig. 2. The spacecraft has an inertia \( I_s \) about the \( \hat{e}_3 \) axis. Two masses \( m_1 \) and \( m_2 \) are attached symmetrically to this craft through two tethers. Initially the body is spinning at a constant rate \( \omega(0) = \omega_0 \hat{e}_3 \). At time \( t_0 = 0 \), the bodies are released from the main spacecraft body. The tethers now begin to unspool from the cylindrical spacecraft body. Assume that the cables will remain tangential to the body surface as shown in Fig. 2. Use the coordinate frames \( S : \{\hat{s}_\phi, \hat{s}_R, \hat{s}_3\} \), \( E : \{\hat{e}_1, \hat{e}_2, \hat{e}_3\} \) and the inertial frame \( N : \{\hat{n}_1, \hat{n}_2, \hat{n}_3\} \). Assuming \( m = 2m_1 = 2m_2 \).

(a) Express the inertial position and velocity vectors \( r_1 \) and \( r_2 \) of both point masses \( m_1 \) and \( m_2 \).

(b) Derive the inertial angular momentum vector \( H \) of the entire system.

(c) Derive the total kinetic energy expression \( T \) for the entire system.

(d) Using conservation laws, find expressions for \( \dot{\phi}(t) \) and \( \omega(t) \) in terms of the initial conditions. Use the parameter \( \gamma = I_s/(mR^2) + 1 \).

(e) Assume we would like the craft to be at rest at time \( t = T \) with \( \omega(T) = 0 \). How much cable length \( l = R\phi \) will you need?

(f) How long \( T \) will it take for the spacecraft hub to be at rest?

Figure 2: Illustration of cylindrical spacecraft with two tethered masses.