Problem 1. Show that the vector product can be written as follows:

\[ \vec{a} \times \vec{b} = [\vec{a}]_\times \vec{b}, \]  

(1)

where \([\vec{a}]_\times\) is the following skew-symmetric matrix formed out of the elements of \(\vec{a}\):

\[
[\vec{a}]_\times = \begin{bmatrix}
0 & -a_3 & a_2 \\
a_3 & 0 & -a_1 \\
-a_2 & a_1 & 0
\end{bmatrix}.
\]

(2)

A skew-symmetric matrix has the property \([\vec{a}]_\times^T = -[\vec{a}]_\times\)

Problem 2. Parameterize the direction cosine matrix \(C\) in terms of (2-3-2) Euler angles, and find appropriate inverse transformations from \(C\) back to the (2-3-2) Euler angles. What are the points of singularity for this representation?

Problem 3. Do textbook problem 3.13 (a),(b), and (c)

Problem 4. Do textbook problem 3.14

Problem 5. For a general orientation, show that the rotation matrix \(C\) can be expressed in terms of the unit quaternion \(\beta\) using the definition of the unit quaternion as following:

\[
\beta = \begin{bmatrix} \beta_0 \\ \vec{\beta} \end{bmatrix}, \quad \beta^T \vec{\beta} = \sin^2 \frac{\Phi}{2}, \quad \beta_0^2 = \cos^2 \frac{\Phi}{2}, \quad [\vec{\beta}]_\times = \sin \frac{\Phi}{2} [\hat{e}]_\times.
\]

Problem 6. Do textbook problem 3.12