3.1 Given three reference frames \( N, B, \) and \( F \), let the unit base vectors of the reference frames \( B \) and \( F \) be
\[
\hat{b}_1 = \frac{1}{3} \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \quad \hat{b}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \hat{b}_3 = \frac{1}{3\sqrt{2}} \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}
\]
and
\[
\hat{f}_1 = \frac{1}{4} \begin{pmatrix} 3 \\ -2 \\ \sqrt{3} \end{pmatrix}, \quad \hat{f}_2 = \frac{1}{2} \begin{pmatrix} -1 \\ 0 \\ \sqrt{3} \end{pmatrix}, \quad \hat{f}_3 = \frac{-1}{4} \begin{pmatrix} \sqrt{3} \\ 0 \\ 2\sqrt{3} \end{pmatrix}
\]
where the base vector components are written in the \( N \) frame. Find the direction cosine matrices \([BF]\) that describe the orientation of the \( B \) frame relative to the \( F \) frame, along with the direction cosine matrices \([BN]\) and \([FN]\) that map vectors in the \( N \) frame into respective \( B \) or \( F \) frame vectors.

3.2 Assume a body frame \( B \) has its orientation defined through \( \{\hat{b}_1, \hat{b}_2, \hat{b}_3\} \). Further, assume that these three \( \hat{b}_i \) unit vectors are given with vector components taken in the \( N \) frame. Show that the direction cosine matrix \([BN]\) can be expressed as
\[
[BN] = \left[ \begin{array}{c} (N\hat{b}_1)^T \\ (N\hat{b}_2)^T \\ (N\hat{b}_3)^T \end{array} \right] = \left[ \begin{array}{c} (B\hat{n}_1)^T \\ (B\hat{n}_2)^T \\ (B\hat{n}_3)^T \end{array} \right]
\]

3.4 Let the vector \( v \) be written in \( B \) frame components as
\[
^Bv = 1\hat{b}_1 + 2\hat{b}_2 - 3\hat{b}_3
\]
The orientation of the \( B \) frame relative to the \( N \) frame is given through the direction cosine matrix
\[
[BN] = \begin{bmatrix} -0.87097 & 0.45161 & 0.19355 \\ -0.19355 & -0.67742 & 0.70968 \\ 0.45161 & 0.58065 & 0.67742 \end{bmatrix}
\]
(a) Find the direction cosine matrix \([NB]\) that maps vectors with components in the \( B \) frame into a vector with \( N \) frame components.
(b) Find the \( N \) frame components of the vector \( v \).

3.5 Using the direction cosine matrix \([BN]\) in problem 3.4, find its real eigenvalue and corresponding eigenvector.