Fast Inertia Property Estimation via Convex Optimization for the Asteroid Redirect Mission

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Abstract— Accurate inertia property estimation is critical to the success of the upcoming asteroid redirect mission. The inertia tensor, center of mass, and total mass of the spacecraft-asteroid combined rigid body must be accurately estimated so that solar electric propulsion can be used to redirect an asteroid into an orbit around the Earth. This paper develops an efficient algorithm to solve for those properties. The estimation is framed as a least squares minimization problem subject to convex constraints. A standard least squares approach is not sufficient due to a matrix rank deficiency arising from the fact that a pure torque cannot be applied to the asteroid after capture. The constrained least squares minimization framework allows for fast inertia estimation with a convex optimization solver, in the sense that accurate estimates can be made with only a few force inputs and response measurements. Simulations are performed in MATLAB R2013B using the CVX 2.1 convex optimization solver to assess the algorithm's performance in a typical mission scenario.

I. INTRODUCTION

The proposal for an asteroid redirect mission dates back over 100 years, but only recently have sufficiently capable solar electric propulsion (SEP) systems been developed to make the mission feasible [1]; see Figure 1.

SEP thrusters, while highly efficient in deep space, can produce only a fraction of the thrust of traditional chemical rockets. As a result, SEP thrusters must fire for many hours, if not days, to achieve the desired change in a spacecraft's velocity. This propulsion characteristic makes it difficult to design an optimal interplanetary trajectory, especially after capturing an asteroid. This is because SEP-based trajectories rely on thousands of carefully timed and calibrated thrust adjustments. Such an open-loop control system hinges on accurate knowledge of the spacecraft's inertia properties.

When the asteroid is captured by the spacecraft, the spacecraft's inertia tensor, center of mass (CM), and total mass change dramatically. Additionally, the process of capturing the asteroid will likely result in residual motion requiring de-tumbling. De-tumbling relies on accurate new inertia tensor and shifted CM estimates to properly torque the spacecraft via an optimal control methodology. For these reasons, there is a need to develop a high performance estimation algorithm to accurately estimate the inertia properties of the combined spacecraft-asteroid rigid body. The accuracy of these estimates is particularly important considering the asteroid will be brought back towards the Earth at high velocity.

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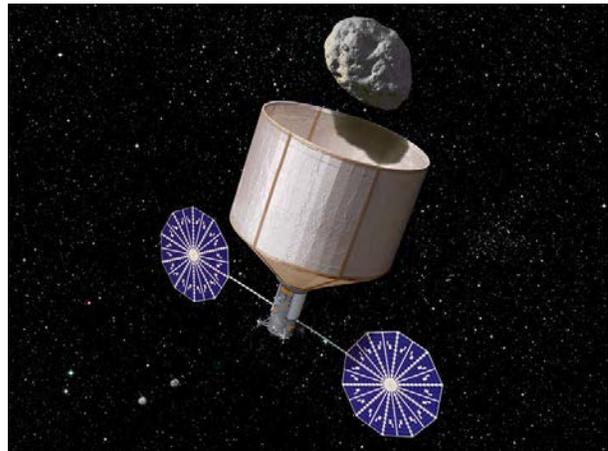
Fig. 1. Illustration of an SEP spacecraft capturing an asteroid; Courtesy of "Asteroid Retrieval Feasibility Study" by Keck Institute for Space Studies, 2012.

The standard approach in this situation is to approximate the solution using least squares. However, due to the fact that the CM shifts to an unknown location after capture, it is imperative to know if a force applied to the combined-body will result in the desired rotation or translation. This paper develops an efficient convex optimization algorithm to estimate the spacecraft-asteroid inertia properties.

Least squares algorithms for identifying mass properties have been studied in various applications, such as spinning vehicles during coasting maneuvers [2], controlling three-thruster vehicles with on-line gyroscopic signals [3], [4], and controlling rigid body links of manipulators [5], [6]. The least squares problem has also been represented as a convex quadratic program which can be efficiently solved with a guaranteed convergence time. An estimation algorithm exploiting torque-free motion is proposed in [7] to approximate the inertia ratios for a tumbling satellite. Examples of the data used for weighted least squares estimation of this type are attitude and rate telemetry measurements [8], [9] and video from on-orbit cooperative vision sensors [10].

It is also possible to improve the performance of the least squares algorithm with additional convex constraints. In [11], a constrained least squares method uses lower and upper bounds on the inertia tensor to improve spacecraft inertia estimation. In the case of the asteroid redirect mission, we can further constrain the estimated parameters based on known physical properties, such as the positive definiteness of the inertia tensor and the restricted location of the CM based on the geometric configuration of the asteroid and spacecraft. Based on that reasoning we have formulated the

estimation algorithm as a least squares minimization problem subject to convex constraints.

The rest of the paper is organized as follows. In §II, we present equations of motion for a spacecraft in deep space and discuss the unknown parameters to be estimated. We then present an estimation procedure in §III. First, we arrange the problem into standard least squares form and show the standard problem setup cannot be applied. Next, we propose convex constraints on the unknown inertia parameters and present a constrained least squares algorithm. In §IV, we explore numerical simulations of the proposed method. Finally, concluding remarks and possible future extensions of this work are presented in §V.

II. EQUATION OF MOTION

In this section, we present a set of governing equations for translational and rotational motion of a spacecraft. We first presume that this estimation process takes place in deep space and the gravity of the asteroid is negligible. In addition, the spacecraft is assumed to be fully controllable and observable in 3 translational degrees of freedom (DOF) but we restrict the spacecraft from making any pure rotation maneuvers. When the location of the shifted CM is unknown, it is not possible to guarantee that a pure rotation generated in the spacecraft frame would cause pure rotation in the combined frame. Therefore the only relevent inputs needed from the spacecraft are those that cause translation in the spacecraft frame.

After capturing the asteroid, a force will be applied by the spacecraft to estimate the inertia properties of the combined-body. The corresponding rigid body motions are governed by the inertial time derivatives of translational and angular momenta represented in the combined-body frame attached to the shifted center of mass; see Fig. 2 for a diagram of an asteroid captured by a spacecraft. A set of differential equations are derived as follows [12]:

$$\left[\frac{d}{dt}(mv)\right]_{\mathcal{B}} = m\dot{v}_b + \omega_b \times mv_b = F, \tag{1}$$

$$\left[\frac{d}{dt}(J\omega)\right]_{\mathcal{B}} = J\dot{\omega}_b + \omega_b \times J\omega_b = (d+r) \times F, \tag{2}$$

where $[d(\cdot)/dt]_{\mathcal{B}}$ denotes an inertial time derivative represented in the shifted body frame $\mathcal{B}$ and $m \in \mathcal{R}$ denotes the mass of the rigid body comprised of the asteroid and spacecraft. The variable $J \in \mathcal{S}_{++}^3$ denotes a positive definite inertia tensor represented in the shifted body frame defined as

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{12} & J_{22} & J_{23} \\ J_{13} & J_{23} & J_{33} \end{bmatrix}, \tag{3}$$

and the vector r denotes a position vector, represented in the combined-body frame, from the center of mass of the spacecraft to the reaction thruster and d denotes the position vector between the center of mass of the combined-body and the center of mass of the spacecraft. The angular and translational velocities are represented by $\omega_b, v_b \in \mathcal{R}^3$ in
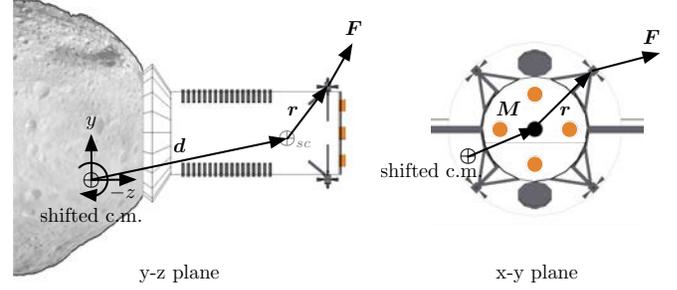

Fig. 2. Diagram of the spacecraft with an asteroid

the combined-body frame. And finally, F denotes the force applied by the spacecraft, written in the combined-body frame; see the illustration in Fig. 2. Due to the position and orientation of F relative to the shifted center of mass, F will also induce the torque $(d + r) \times F$. There are several methods for accurate characterization of $\omega_b, v_b, \dot{\omega}, \dot{v}_b$ and F on-board in an operational spacecraft, including: accelerometers, gyroscopes, star-tracking optical sensors, and video from secondary spacecraft or Earth-based telescopes [3], [10]. In this paper we assume perfect knowledge of $\omega_b, v_b, \dot{\omega}, \dot{v}_b$ and F.

Thus, the unknown parameters we will estimate are,
- combined inertia tensor (asteroid + spacecraft) J
- location of the shifted center of mass d
- combined mass (asteroid + spacecraft) m

Note that in order to focus on the feasibility of the estimation algorithm, we assume that all external disturbances on the spacecraft, such as gravitational forces and solar radiation pressure, are negligible.

III. ESTIMATION PROCEDURE

In this section, we estimate the inertia properties using a least squares method. The measurement data are substituted into the rotational and translational dynamics such that the estimated inertia property values minimize the Euclidean-norm error.

1) Least squares: The standard setup for a least squares problem is the solution of a system of linear equations,

$$Ax = b \tag{4}$$

where $x \in \mathcal{R}^n$ denotes a vector of unknown parameters, $A \in \mathcal{R}^{m \times n}$ denotes a regressor matrix with m independent rows (measurements in our case), and $b \in \mathcal{R}^m$ denotes a vector containing known parameters. For a specific x, the residual error e is defined by

$$e = Ax - b. \tag{5}$$

We can find a solution x that minimizes the sum of squares of the residual error given by

$$H = \frac{1}{2}(Ax - b)^T(Ax - b) = \frac{1}{2}e^T e. \tag{6}$$

Since H is convex in x, the global minimum is found at the point where the gradient of the objective function equals zero

$$\nabla H = A^T A x - A^T b = 0. \tag{7}$$

Thus, we obtain the solution vector x for least squares as

$$x = (A^T A)^{-1} A^T b, \tag{8}$$

where the measured data b should not be the zero vector, as this would return a trivial all-zero solution for x. Note that in order to have a unique solution, the *Hessian* of H should be positive definite [13], i.e.,

$$\nabla^2 H = A^T A \succ 0. \tag{9}$$

Now let us use the standard setup for an inertia estimation problem. By defining $x_J \in \mathcal{R}_{++}^6$:

$$x_J = [J_{11}, J_{22}, J_{33}, J_{12}, J_{13}, J_{23}]^T. \tag{10}$$

the left hand side of Eq. (2) can be rewritten as

$$J\dot{\omega}_b + \omega_b \times J\omega_b = ([\dot{\omega}]_* + [\omega]_\times [\omega]_*)x_J, \tag{11}$$

where given $\omega = [\omega_1, \omega_2, \omega_3]$, $[\omega]_\times$ is defined by the cross product operator as

$$[\omega]_\times = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}, \tag{12}$$

and $[\omega]_*$ is defined as

$$[\omega]_* = \begin{bmatrix} \omega_1 & 0 & 0 & \omega_2 & \omega_3 & 0 \\ 0 & \omega_2 & 0 & \omega_1 & 0 & \omega_3 \\ 0 & 0 & \omega_3 & 0 & \omega_1 & \omega_2 \end{bmatrix}. \tag{13}$$

Then the rigid body dynamics, Eqs. (1),(2) can be written in the form of the least squares problem given in Eq. (4) with measurement data at time t as

$$A(t) = \begin{bmatrix} [\dot{\omega}]_* + [\omega]_\times [\omega]_* & [F]_\times & 0_{3\times 1} \\ 0_{3\times 6} & 0_{3\times 3} & \dot{v} + [\omega]_\times v \end{bmatrix}_{6\times 10}, \tag{14}$$

with

$$x = \begin{bmatrix} x_J \\ d \\ m \end{bmatrix}_{10\times 1} \quad \text{and} \quad b(t) = \begin{bmatrix} [r]_\times F + M \\ F \end{bmatrix}_{6\times 1}, \tag{15}$$

where d denotes a position vector joining the shifted CM and the spacecraft CM and m denotes the total mass.

Thus, the n measurements constitute the least squares parameter estimation problem,

$$Ax = b, \tag{16}$$

where $A \in \mathcal{R}^{6n\times 10}$ and $b \in \mathcal{R}^{6n}$ given by

$$A = \begin{bmatrix} A(1) \\ \vdots \\ A(n) \end{bmatrix}, \quad b = \begin{bmatrix} b(1) \\ \vdots \\ b(n) \end{bmatrix}. \tag{17}$$

Then a typical least squares method can be carried out to obtain a solution vector x as follows:

$$x = (A^T A)^{-1} A^T b. \tag{18}$$

Note that given the dynamics Eqs. (1)-(2) and 10 estimation variables, we require

$$\text{Rank}(A^T A) = 10, \tag{19}$$

in order to ensure the condition $A^T A \succ 0$ in Eq. (9). Since all measurements are not assumed to be linearly independent, we need many measurements to improve the method's accuracy. However, under the proposed restriction that the spacecraft can only apply translational forces, $A^T A$ cannot be full rank regardless of the number of linearly independent measurements. This means that the standard least squares approach will be unsuccessful. We explain this in the following proposition.

Proposition 1: Given dynamical models which only allow translational maneuvers to explore the rigid body's configuration space, the standard least squares approach cannot carry out the optimal estimation during an estimation process as

$$\max_n \text{Rank}(A^T A) \leq 9. \tag{20}$$

Proof: With n measurements, the sufficient condition to have a unique solution, Eq. (9), can be computed as

$$\nabla^2 H = A^T A = \begin{bmatrix} L_{6\times 6} & M_{6\times 3} & 0_{6\times 1} \\ M_{3\times 6}^T & N_{3\times 3} & 0_{3\times 1} \\ 0_{6\times 1} & 0_{3\times 1} & P_{1\times 1} \end{bmatrix}_{10\times 10} \tag{21}$$

with

$$L_{6\times 6} = \sum_i^n ([\dot{\omega}]_* + [\omega]_\times [\omega]_*)^T ([\dot{\omega}]_* + [\omega]_\times [\omega]_*), \tag{22}$$

$$M_{3\times 6} = \sum_i^n ([\dot{\omega}_i]_* + [\omega_i]_\times [\omega_i]_*)^T [F_i]_\times, \tag{23}$$

$$N_{3\times 3} = \sum_i^n [F_i]_\times^T [F_i]_\times, \tag{24}$$

$$P_{1\times 1} = \sum_i^n [\dot{\omega}_i]_*^T [\dot{\omega}_i]_* + 2[\dot{\omega}_i]_*^T ([\omega_i]_\times [\omega_i]_*) \tag{25}$$

$$+ ([\omega_i]_\times [\omega_i]_*)^T ([\omega_i]_\times [\omega_i]_*), \tag{26}$$

where $i = 1, ..., n$ denotes the time index for the measurement. Let $x_p = [x_J^T, (d+r)^T, 0]^T$. The quadratic form on the *Hessian* yields

$$x_p^T \nabla^2 H x_p = x_J^T L x_J + (d+r)^T N(d+r)$$
$$+ (d+r)^T M^T x_J + x_J^T M(d+r)$$
$$= \sum_i^n \Big[\|([\dot{\omega}_i]_* + [\omega_i]_\times [\omega_i]_*)x_J\|^2 + \|[F_i]_\times (d+r)\|^2$$
$$+ 2(([\dot{\omega}_i]_* + [\omega_i]_\times [\omega_i]_*)x_J)^T ([F_i]_\times (d+r)) \Big], \tag{27}$$

and the last equation is the 2-norm square of Eq. (2) as

$$\|J\dot{\omega}_b + \omega_b \times J\omega_b - (d+r) \times F\|^2 = 0. \tag{28}$$

Thus, we have

$$x_p^T \nabla^2 H x_p = 0,$$

and regardless of the number of measurements, H is not full rank. ∎

Such unobservable states due to the rank deficiency of the regressor matrix can be addressed by enforcing constraints under a convex programming framework. We address such an approach in the following section.

2) Constrained least squares: The least squares problem can be represented as a convex quadratic program which can be efficiently solved with a guaranteed convergence time. This approach is also desirable because the original least squares problem cannot be solved by an analytical solution when inequality constraints are added. Our algorithm relies on the property that the resulting problem is convex when these constraints are convex [14].

We can find constraints on the estimated parameters x from known physical properties as well as the geometric configuration between the asteroid and the spacecraft. For example, one obvious constraint is that the combined mass m is positive. Even such a minute specification leads to computational improvements which lower the required number of measurements needed to meet the desired accuracy; see §IV. We can find more physical properties of the inertia tensor from its definition.

Proposition 2: The inertia tensor J is a positive semidefinite matrix and the sum of two of its diagonal components is always larger than the third diagonal component, i.e.,

$$J_i + J_j \geq J_k, \tag{29}$$

where J_i denotes the entry on the ith row and ith column of the inertia tensor J.

Proof: The inertia tensor can be reformulated as

$$\boldsymbol{J} = \int \left(\|\boldsymbol{r}\|^2 \mathbf{I}_3 - \boldsymbol{r}\boldsymbol{r}^T \right) dm. \tag{30}$$

This forms a symmetric matrix and it is easily observed that its eigenvalues are all nonnegative since

$$\text{eig} \left(\boldsymbol{r}\boldsymbol{r}^T \right) = 0, 0, \|\boldsymbol{r}\|^2. \tag{31}$$

Thus, $\boldsymbol{J}$ is positive semidefinite. Moreover, by definition, we have $J_1 = \int (y^2 + z^2)dm$, $J_2 = \int (x^2 + z^2)dm$, $J_3 = \int (x^2 + y^2)dm$. Since integration is a linear operation, we can find the relationship between J_1 and J_2 as follows:

$$J_1 + J_2 = \int (y^2 + 2z^2 + x^2)dm$$
$$\geq J_3 = \int (x^2 + y^2)dm \tag{32}$$

and we obtain $J_1 + J_3 \geq J_2$ and $J_2 + J_3 \geq J_1$ in the same way. ∎

The center of mass is an important parameter for precise translational control. Using the fact that the shifted CM
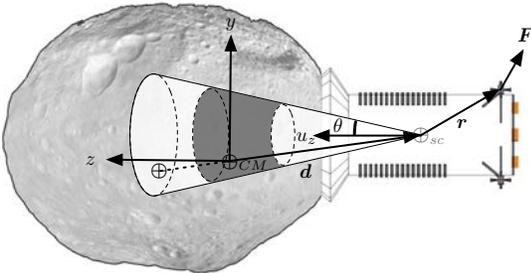

Fig. 3. Convex cone for the shifted center of mass

always lies on the line between the CM of the asteroid and the CM of the spacecraft, we can find bounds on the position of the shifted CM. Since the CM of the asteroid is located inside asteroid's physical boundary, the shifted center of mass exists in a convex cone as illustrated in Fig. 3. This cone can be tapered further if we obtain tighter bounds on the location of the asteroid's CM.

Proposition 3: The shifted center of mass is always positioned in the shaded zone depicted in Fig. 3 and this forms a convex hull of all feasible centers of mass. Such a convex set is represented as:

$$\{d \in \mathcal{R}^3 \,|\, d^T u_z \leq -\|d\| \cos\theta \text{ and } \delta_l \leq d_z \leq \delta_u\} \tag{33}$$

where $d = [d_z, d_y, d_z]^T$ and u_z denotes the unit vector along the z-axis in the body frame.

Proof: The condition that the position vector stays within an angle θ around the unitized z body axis can be represented by the dot product as

$$-d^T u_z \geq \|d\|\|u_z\| \cos\theta, \tag{34}$$

where $\|u_z\| = 1$ and d denotes a vector from the shifted CM to the CM of spacecraft before capture. Then, we have

$$f(d) = d^T u_z + \|d\| \cos\theta \leq 0. \tag{35}$$

Since $f(d)$ is a linear combination of an affine function and Euclidean norm, it is convex. ∎

Taking advantage of all known properties of the inertia tensor J, namely that J is positive definite and that the sum of its two smaller eigenvalues should be larger than or equal to its largest eigenvalue, the overall constrained least squares problem can be written as

$$\text{minimize} \quad \frac{1}{2}\|\boldsymbol{A}x - \boldsymbol{b}\|_2^2 \tag{36}$$
$$\text{subject to} \quad J \succeq 0 \tag{37}$$
$$\lambda_1(J) \leq \lambda_2(J) + \lambda_3(J) \tag{38}$$
$$m \geq 0. \tag{39}$$

By defining the state vector x as in Eq. (10), the inertia tensor estimate naturally forms a symmetric matrix. However, with its 6 elements, developing a positive semi-definiteness constraint results in a non-convex constraint. Thus, we relax the positive semi-definiteness of the inertia tensor as follows:

$$J \succeq 0 \quad \Rightarrow \quad J_1, J_2, J_3 \geq 0 \tag{40}$$

Rewriting this with all convex constraints yields

$$\text{minimize} \quad \frac{1}{2}\|\boldsymbol{A}x - \boldsymbol{b}\|_2^2 \tag{41}$$

subject to

$$\begin{aligned}
J_i \geq \alpha_i, \quad i = 1, 2, 3 & \quad \}① \\
J_1 \leq J_2 + J_3 & \\
J_2 \leq J_3 + J_1 & \quad \}② \\
J_3 \leq J_1 + J_2 & \\
d^T u_z + \|d\| \cos\theta \leq 0 & \quad \}③ \\
\delta_l \leq d_z \leq \delta_u & \quad \}④ \\
m \geq \beta_m & \quad \}⑤
\end{aligned}$$
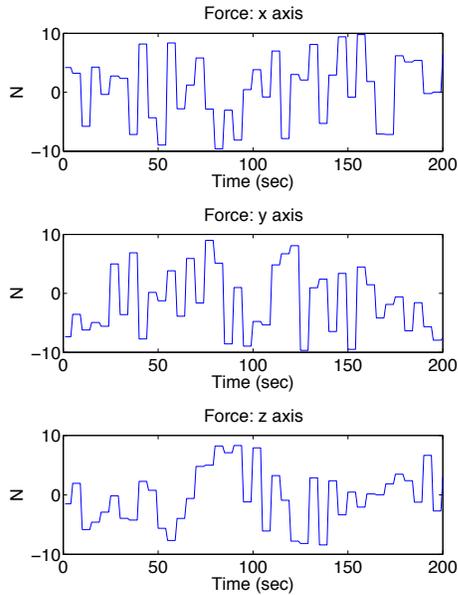
Fig. 4.   Time histories of applied forces

Parameters	Values
True Inertia tensor, $\boldsymbol{J}$	$\begin{bmatrix} 57928 & -7568 & -9744 \\ -7568 & 108692 & 7675 \\ -9744 & 7672 & 153069 \end{bmatrix} kg \cdot m^2$
Asteroid mass, m_a	$625000\ kg$
Spacecraft mass, m_s	$7000\ kg$
Total mass, m	$632000\ kg$
$\boldsymbol{r}$	$[\,-1.04 \quad 3.12 \quad -6.763\,]^T\ m$
d	$[\,0.9 \quad 2.1 \quad -12\,]^T\ m$
Force, F	$-10\ N \leq F \leq 10\ N$
v_0	$[\,0.12 \quad -0.29 \quad 0.26\,]^T\ m/s$
ω_0	$[\,0.15 \quad -0.12 \quad 0.31\,]^T\ rad/s$
Accuracy	10^{-1}

TABLE I

SIMULATION PARAMETERS

Parameters	Values
δ_l	-16
δ_u	-5
Convex cone angle, θ	9°
u_z	$[\,-0.073 \quad -0.17 \quad 0.98;]^T$
$[\,\alpha_1, \alpha_2, \alpha_3\,]$	$[\,30000, \quad 60000, \quad 100000\,]kg \cdot m^2$
β_m	$400000\ kg$

TABLE II

PARAMETERS NEEDED TO DEFINE THE CONVEX CONSTRAINTS

Additional constraints such as lower and upper bounds of the inertia tensor can be incorporated into this framework to improve the convergence rate. However, it is difficult to find these lower and upper bounds because the inertia tensor is sensitive to small changes in the shifted CM and total mass, both of which are unknown.

IV. SIMULATIONS

In this section, we conduct several simulations to compare the inertia property estimation methods presented above. The simulations are carried out in MATLAB R2013b using the CVX 2.1 convex optimization solver. We assume a 7 metric ton SEP-spacecraft captures a 625 ton asteroid in deep space. The translational and angular velocities after the capture are presented in Table I. Before estimation begins, the combined-body is assumed to have an initial translational velocity v_0 and rotational velocity ω_0. For the estimation procedure, we assume the spacecraft's reaction thruster system generates a sequence of forces in alternating directions. In order to minimize drift in position and orientation of the combined-body caused by the applied force, we reverse the force direction every 5 seconds; see Fig. 4. The thrust in each direction is randomly selected within $\pm 10N$ in order to encourage linearly independent measurements. We measure the translational accelerations including simulated noise every second. Angular and translational velocities are assumed to be obtained by seamless integrations. We note that the reaction thruster system is configured to be symmetric along the z-axis of the spacecraft as illustrated in Fig. 2. The simulated time histories for angular and translational velocities are depicted in Fig. 5.

We first run the simulation using only the least squares formulation with narrower z-axis bounds on the shifted CM. Note that since the regressor matrix A is rank deficient, at least one constraint is required to find a feasible solution. Furthermore, as seen in Figs. 6 and 7, for the first 70

estimations, the error does not decrease. This is because it takes time for the independent bases to fill the convex constrained configuration subspace.

The error is calculated by the l^2-norm with a tolerance of 10^{-1}. The inertia tensor estimation converges to the designated accuracy level within 200 measurements when all constraints are applied. Adding the inequality constraint ⑤ in Eq. (41) that the total mass has a lower bound also reduces the required number of measurements as seen in Fig. 6. Such a lower bound could be obtained through remote observation of the asteroid. Applying the constraint for convex inertia tensor properties ① lowers the requirement by about 50 measurements as represented in Fig. 6.

After 100 and 200 measurements, the estimates of shifted inertia tensors are the following:

$$J_{100} = \begin{bmatrix} 52247 & -6825.7 & -8788.2 \\ -6825.7 & 98031 & 6917.5 \\ -8788.2 & 6917.5 & 138065 \end{bmatrix} kg/m^2 \quad (42)$$

$$J_{200} = \begin{bmatrix} 57625 & -7528.3 & -9692.9 \\ -7528.3 & 108122 & 7629.8 \\ -9692.9 & 7629.8 & 152275 \end{bmatrix} kg/m^2, \quad (43)$$

When estimating the CM, restoring a position vector d from Eq. (1) is not possible since the cross product is not invertible. However, the correlation matrix $\boldsymbol{A}^T \boldsymbol{A}$ along with the extra convex constraints on states enable us to estimate the location of the CM precisely. As discussed earlier, the
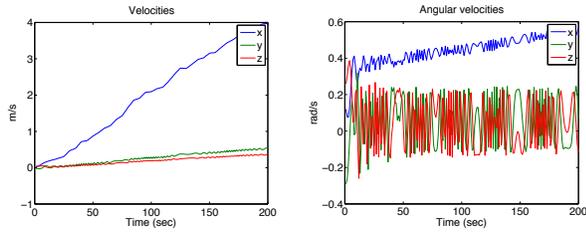
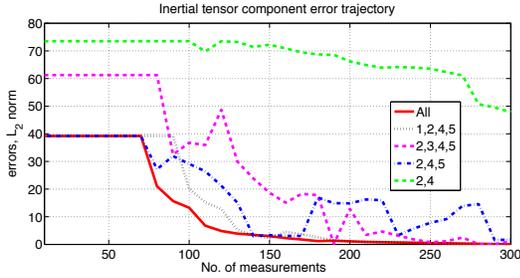Fig. 5. Time histories of angular and translational velocities of the combined-body

Fig. 6. Inertial tensor component error trajectory over number of measurements

standard least squares method cannot be applied in this scenario. As seen in Fig. 7, the convex cone constraint ③ diminishes the requirement dramatically; see plots for (②, ③, ④, ⑤) and (②, ④, ⑤). The parameters defining this convex cone are presented in Table II. The inertia tensor constraint ① reveals that it also helps to reduce the number of measurements.

Note that the estimation of the inertia properties depends on 9 linearly independent measurements and the corresponding quotient space can be identified by the measurements' orthogonal complements within a convex constrained domain [15]. Thus, we require translational forces in many independent directions.

V. CONCLUSION

The asteroid redirect mission will be a milestone in humanity's exploration and understanding of the solar system. However, there are many technical challenges to overcome before it is feasible to capture an asteroid and redirect it into an orbit around the Earth. One such challenge is accurately estimating the inertia properties of the spacecraft-asteroid body. The challenge arises because the location of the shifted CM is not known, which makes it impossible to calculate the rotation of the combined-body about the shifted CM. It has
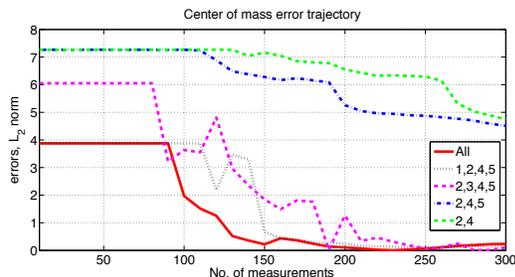

Fig. 7. Center of mass error trajectory over number of measurements

been shown that the standard least squares approach cannot estimate the inertia properties under such circumstances.

We approached this problem by developing an algorithm framed as a least squares minimization problem subject to convex constraints. We simulated the algorithm in MATLAB R2013B using the CVX 2.1 convex optimization solver. As expected, the algorithm converged with fewer measurements as we supplied it with more convex constraints. This is an encouraging result because any future improvements in the estimates of the asteroid's inertia properties can be easily incorporated into our algorithm as new or refined constraints.

Future research related to this topic involves adding more realism to the simulation. It remains to be seen how solar and gravitational disturbances, minute thruster misalignment, a deformable spacecraft model, and other uncertainties impact the proposed approach. It will also be useful to reformat the algorithm so that it can work during the mission's return trajectory. An asymmetric asteroid shifting mid-flight will have disastrous consequences for the SEP open-loop control profile unless the spacecraft has the ability to recalculate the inertia properties while transporting the asteroid.

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