Deconfliction Algorithms for a Pair of Constant Speed Unmanned Aerial Vehicles

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Abstract—This paper provides an approach for the deconfliction problem for a pair of constant speed unmanned aerial vehicles (UAVs) modeled as unicycles in presence of static constraints. We use a combination of navigation and swirling functions to direct the unicycle vehicles along the planned trajectories while avoiding inter-vehicle collisions. The main feature of our contribution is proposing means of designing a deconfliction algorithm for unicycle vehicles that more closely capture the dynamics of constant speed UAVs as opposed to double integrator models. Specifically, we consider the issue of UAV turn-rate constraints and proceed to explore the selection of key algorithmic parameters in order to minimize undesirable trajectories and overshoots induced by the avoidance algorithm. The avoidance and convergence analysis of the proposed algorithm is then performed for two cooperative UAVs and simulation results are provided to support the viability of the proposed framework for more general mission scenarios.

Index Terms—Collision avoidance, UAV deconfliction, Unicycle, Decentralized Control

I. INTRODUCTION

Modern technology allows flying groups of unmanned aerial vehicles (UAVs) to perform many civilian and military missions, including surveillance, monitoring, imaging, and reconnaissance. Collision avoidance and deconfliction become of paramount importance when such missions require that UAVs operate in close proximity of each other.\(^1\) It has thus become necessary to develop efficient algorithms to ensure that these vehicles stay conflict-free while completing their intended missions. There has been a number of research works on conflict and collision avoidance between autonomous vehicles; see [1] for a survey. For example in [2]–[4] the authors propose a prescribed set of maneuvers and protocols for each vehicle when a conflict has been detected. In [5], probabilistic method is used to for noncooperative collision avoidance. The work of [6] involves a heuristic search for a tree of feasible avoidance maneuver using velocity obstacles. Optimization-based schemes on the other hand, as exemplified in [7], [8], revolve around solving optimization problems, often in centralized way, to resolve conflicts, either deterministically or in a probabilistic setting. In Ref. [7], and other works on collision avoidance such as Refs. [9]–[12], the notion of collision cone has been introduced for each vehicle, parameterizing the zone where the collision will occur if all other vehicles keep their heading unchanged. The geometric optimization approach in [13] also utilizes the collision cone by minimizing the velocity vector changes such that the relative velocity between two conflicting vehicles stay off the cone.

More close to the present work are approaches that are developed with the aid of a potential or a navigation function. A navigation function landscapes the configuration space of the vehicles such that the goal points for the vehicle group can be reached by following the function’s descent direction. In this venue, in Refs. [14], [15] the dynamics of mobile ground robots were included in the navigation function using the dynamic window approach (DWA), while in Khatib [16] and Khatib and Brock [17], such functions have been adopted for holonomic robots. Rimon and Koditschek [18] on the other hand, introduced a navigation function for robot obstacle avoidance in a generalized “star world” with a static environment. The convergence analysis for the trajectories generated by the navigation function has also been explored in Ref. [19].

Recently, the use of decentralized navigation functions has gained attention in various research works in the area of cooperative and formation control; see for example, Refs. [20], [21]. Being “decentralized” implies that each vehicle does not require knowledge of the states and desired destinations of all vehicles and merely relies on limited information, such as relative position and velocity from the “neighboring” vehicles, to adjust its steering strategy. The decentralization is particularly suited for large number of vehicles as it reduces the computational aspects of gradient computation as well as the associated communication and sensing requirements. In this direction, we mention the works of Dimarogonas and Kyriakopoulos [22], [23] that have utilized the notion of decentralized navigation function for addressing collision avoidance between multiple holonomic agents in the absence of control constraints. Collision avoidance for nonholonomic vehicles has been considered by Aicardi et al. [32], [33], that utilize the velocity vector field and tracking of 2D path to avoid obstacles. The work of Ajith and Ghose [9] utilizes the notion of collision cone to static obstacles where the lateral acceleration commands are derived from the largest possible radii that ensure collision avoidance. The issues of turn-rate limit and tracking the nominal path are also included. Other recent works on collision avoidance include those in [30], as well as [34], [35], the latter adopting a decentralized optimization approach that requires large computational resources. In [31], a kinematic controller and dipolar navigation function has been proposed to avoid collisions and configure nonholonomic agents to the desired orientation at their respective destinations.

Our approach to the unicycle-model UAV deconfliction presented in this paper builds upon the following desirable

\(^1\)Deconfliction refers to collision avoidance with the addition requirement that vehicles stay as close as possible to their original intended path or trajectories.
properties for the UAV trajectories: (1) guaranteed collision-free convergence to the final desired destination for each UAV in presence of static obstacles, (2) ensured proximity to the nominal path for each UAV when permissible—thus making our approach suited for deconfliction rather than pure collision avoidance, (3) guaranteed collision avoidance for the pair of moving UAVs in the absence of other constraints, and finally, (4) guaranteed operation within the maximum turn-rate for each unicycle UAV during the entire mission. The first two requirements necessitate the introduction of a navigation function—combining path planning and obstacle avoidance in order to create the deconfliction control laws. However, the fourth requirement forces us to employ unicycle models for the UAVs with turn-rate constraints in order to more accurately capture the non-holonomic dynamics of UAVs with constant speed. This model has been widely used in a number of works related to collision avoidance for aerial and ground vehicles such as Ref. [9], [10], [27], [31]. In the meantime, the third requirement invites us to consider an extension of the work presented in Ref. [24], where the swirling effect is introduced for double integrator type models in order to break the symmetry and avoid saddle points around the moving obstacles. This has to be done in conjunction with examining the vehicles’ turn-rates, trajectory overshoots, and minimum distances between vehicles during the deconfliction maneuver as consistent with other requirements.

The contribution of this paper is threefold. On one hand, we show how constraints such as UAV turn rate limits and effective range of detection can be taken into consideration in designing navigation-based deconfliction algorithms. Moreover, we theoretically demonstrate how the tuning parameters for the navigation function can be selected such that vehicles’ actuator commands do not exceed prescribed limits without sacrificing deconfliction performance. And lastly, and more importantly, the collision avoidance and the global convergence of the algorithm are analyzed for a scenario consisting a pair of unicycle type UAVs and verified for more general scenario through simulations.\textsuperscript{2}

The rest of the paper is organized as follow. §II provides the model setup for a pair of identical unicycle UAVs and their representative mission scenarios. In §III, the decentralized navigation function is presented followed by the corresponding deconfliction control laws, as well as the way-point steering method. In this section, we also examine the convergence of the UAVs to their respective destinations along with the guaranteed collision avoidance in presence of static obstacles.

\textsuperscript{2}A preliminary version of this work with a more limited scope has been presented in [29].

§IV introduces the swirling function and presents the proof of guaranteed avoidance for a pair of unicycle UAVs using LaSalle’s Invariance Principle. §V provides the analysis of the tuning parameters that effect UAV turn rates during the deconfliction maneuver and proposes a method for improving the algorithm’s performance. §VI illustrates simulations scenarios, with particular attention to UAV turn-rates and trajectory overshoots as guided by the proposed algorithm. Concluding remarks are provided in §VII.
the protected zones that specify distinct levels of safety constraints. In this figure, the i-th vehicle has a collision zone of radius \( \rho_i > 0 \) and a larger protected zone with radius \( \rho_i + \delta_i \), with \( \delta_i > 0 \).

**Definition 2.1:** (Collision): A collision between vehicle \( i \) and \( j \) occurs when their collision zones intersect or \( \| \mathbf{r}_{ij} \| < \rho_i + \rho_j \), where \( \mathbf{r}_{ij} \) is a vector from vehicle \( i \) to \( j \) as measured from the vehicles’ centers of mass.

**Definition 2.2:** (Conflict): Two vehicles \( i \) and \( j \) are considered to be in a conflict or loss of separation if their protected zones overlap, i.e., when \( \| \mathbf{r}_{ij} \| < \rho_i + \rho_j + \delta_i + \delta_j \).

In the meantime, a group of vehicles are said to be safe or conflict free when their respective protected zones do not intersect during a given maneuver.

In order to implement our deconfliction algorithm, we assume that each unicycle has the capability to detect other vehicles within the range \( r_{ss} \). The detection mechanism can be found in [36], [37]. We define the neighborhood set of the i-th vehicle as

\[
N_i \triangleq \{ j \neq i : \| \mathbf{r}_{ij} \| \leq r_{ss} \}.
\]  
(3)

We note that since the set \( N_i \) depends on the inter-vehicle distances, it is essentially a dynamic set.

Our goal in this paper is to design the admissible inputs \( u_i \) and \( v_i \) for each UAV (1) in order to

(A) guarantee collision free convergence to the final destination or way-point in presence of static obstacles,
(B) stay in the proximity of the nominal path when permissible,
(C) guarantee deconfliction with the other UAV in the range of detection \( r_{ss} \) by not entering its protected zone, and
(D) guarantee turn-rates in accordance with (2).

The first two requirements necessitate the introduction of a navigation function that combines both goal and path attraction to create the control law (§III) while the third and the fourth requirements necessitate the use of a swirling function while ensuring acceptable turn-rates, trajectory overshoots, and minimum separation between the UAVs (§IV).

### III. Navigation Functions for UAVs in Presence of Static Obstacles

One of the ingredients for the proposed algorithm is a navigation function for each UAV that utilizes the location of the obstacles as well as its nominal trajectory. Navigation functions are developed from the intuition that the trajectory of a dynamical system asymptotically approaches a state corresponding to the minimum potential if it follows the descent direction of the potential. As detailed in the introduction, this approach has a long tradition in robotics and aerospace community. In the context of this paper, the navigation function of each UAV is a mapping from the product of the free configuration space for each UAV, denoted by \( \mathcal{F}_i \), to a real-valued potential, as

\[
V_i : \mathcal{F} \to \mathbb{R},
\]  
(4)

where \( \mathcal{F} = \mathcal{F}_i \times \mathcal{F}_j \). In our case, the free configuration space for each vehicle will be the original spherical configuration space \( \mathcal{C}_i \) with removed regions representing static obstacles. Let \( \mathbf{q}_i(t) = [x_i(t), y_i(t), z_i(t)]^T \) represent the position of the i-th vehicle at time \( t \) and let \( \mathbf{q}(t) \) consist of the position vectors for the vehicle pair. The navigation function leads to a mechanism for steering the pair of UAVs from the initial state \( \mathbf{q}_o \) to the destination \( \mathbf{q}_d \) while staying in the product of their respective free configuration spaces. In this direction, each UAV is steered using the negative gradient of the underlying potential function by setting its desired velocity as

\[
v_i^d(t) = -\nabla_{\mathbf{q}_i} V_i(q).
\]  
(5)

Such a velocity guidance, in turn, will lead the UAV to travel in a direction such that the potential is lowered and subsequently reaches its minimum. Therefore a navigation function should have a unique minimum at a destination state \( \mathbf{q}_d \) and a maximum at the boundary of \( \mathcal{F} \). Other properties are also required to guarantee desirable trajectories.

**Definition 3.1:** Let \( \mathbf{q}_d \in \mathcal{F}_i \) be the goal point for UAV \( i \). A real-valued map \( V_i : \mathcal{F} \to \mathbb{R} \) is a navigation function for UAV \( i \) if for any fixed \( \mathbf{q}_i \), as a function of \( \mathbf{q}_i \), it is

(a) a Morse function (its critical points are non-degenerate),
(b) smooth on \( \mathcal{F}_i \) (at least a twice differentiable function),
(c) polar at \( \mathbf{q}_d \) (is a unique minimum), and
(d) admissible on \( \mathcal{F}_i \), i.e., it is uniformly maximal on the boundary of \( \mathcal{F}_i \).

With these properties intact, specific navigation functions can be designed for many robotic path planning applications.

In [18] the navigation function is constructed based on a static environment, where the proof of convergence to \( \mathbf{q}_d \) for each i while avoiding collisions with static obstacles is examined. In this work, we adopt the approach in [18] to construct one of the components of the proposed algorithm for UAV trajectory planning. It is important to note that the collision avoidance among the pair of moving vehicles is not guaranteed using the machinery developed in [18]; in subsequent sections, we introduce the so-called “swirling function” to address this latter issue. The form of the navigation function employed in our work for the i-th vehicle is parameterized as [18],

\[
V_i(q) = \frac{\Gamma_i(q)}{\sqrt{\Gamma_i^2(q) + \beta_i(q)}}
\]  
(6)

where,

- \( \Gamma_i(q) \): is the path planning function which includes goal and path attraction terms, and reaches a unique minimum when the i-th vehicle is at its goal,
- \( \beta_i(q) \): is the obstacle function that is designed to fade out when the i-th vehicle is about to enter the other vehicles protected zone or collide with static obstacles, and
- \( k \) is a positive tuning parameter.

In the work of Rahmani et al. [24], the function \( \Gamma_i \) has been chosen by adding the distance from the goal to the nominal path \( d_i \) weighted by a parameter \( K_p \). This function provides the necessary emphasis between goal and path attraction, i.e.,

\[
\Gamma_i(q) = \| q_i - q_d \|^2 + K_p \beta \left( \frac{\| q_i - q_d \|}{r_p} \right)^2 d_i^2,
\]  
(7)

where
and $\beta$ is a smooth function that transitions from zero to one as defined subsequently (see (12) below); it aims to dim down the path attraction component once the vehicle gets closer than $r_p$ to the goal. Note that $\Gamma_i(q)$ is continuously differentiable on $\mathcal{F}$. In order to prevent two or more vehicles from converging to the same point and at the same time, while also trying to avoid each other, the following assumption for the initial and goal points has been adopted:

**Assumption 3.1:** The initial and final positions for each vehicle is separated by the range of detection $r_{ss}$, i.e.,

$$\|q_{i_0} - q_{o_0}\| > r_{ss} \quad \text{and} \quad \|q_{d_k} - q_{d_j}\| > r_{ss}.$$

(9)

For the case with more than two way-points or when each vehicle has different number of way-points, this assumption should only be applied to all of initial points and the last way-points. This will be discussed further in §III-B. Moreover, as we would like the vehicles to arrive at their respective destinations around the same time, we assume that the initial and final positions of each vehicle is such that the total desired path lengths traversed by the vehicles are equal, i.e.,

$$\text{for all } i \neq j, \|q_{d_i} - q_{o_i}\| = \|q_{d_j} - q_{o_j}\|.$$ (10)

For missions with more than two way-points, we assume that the overall desired path lengths traversed by the vehicles are the same.

The “obstacle” function $\beta_i(q)$ is constructed from the product of pairwise functions of vehicles in $\mathcal{N}_i$ as well as the static obstacles.

Figure 2 depicts the free configuration space which is a sphere of radius $\rho_c$ centered at $q_c$; in this case,

$$\beta_i(q) = \beta(r_{ic}), \prod_{j \in \mathcal{N}_i \cup \mathcal{S}} \beta(r_{ij}),$$

(11)

where $\mathcal{S}$ represents the static obstacles and $r_{ij} = \|q_i - q_j\| - \rho_j$, $r_{ic} = \|q_i - q_c\|$. In the meantime, the bump function has the form

$$\beta(r_{ij}) = \begin{cases} 
0 & r_{ij} < 0, \\
 f(r_{ij}) & 0 \leq r_{ij} < 1, \\
1 & r_{ij} \geq 1,
\end{cases}$$

(12)

where $f(r_{ij})$ is a high order polynomial, as shown in Figure 3, that smoothly transitions the obstacle function from uniformly zero at the collision zone boundary, to one at the protected zone boundary; the parameter $\delta_c$ measures the distance between these boundaries and $\delta_c$ measures the protected distance to the boundary of the configuration space. The polynomial is chosen such that $\beta(q)$ is twice differentiable. It is noted that the choice of the bump function effects the turn-rate of the vehicles during collision avoidance; this will be discussed in more details in §V-C. The configuration radius $\rho_c$ is chosen to be large enough such that $q_{o_i}, q_{d_j}$, and UAVs’ trajectories stay away from the configuration boundary by more than $\delta_c$. Since $\beta(r_{ij})$ is always one when $q_i$ is outside $q_j$’s protected zone, we do not need to include vehicles or static obstacles that are further away from vehicle $j$ in order to compute the function $\beta$.

For each $q_i$, at the boundary of $\mathcal{F}_i$, $\beta_i(q) = 0$ and the function (6) uniformly reaches its maximum of one; on the other hand when $q_i = q_{d_i}$, $\Gamma_i(q) = 0$, and the potential reaches its unique minimum of zero. The function is twice differentiable on $\mathcal{F}$ with the range being the unit interval. The parameter $k$ changes the slope of the navigation function in a way that emphasizes obstacle avoidance or attraction toward the goal, as shown in Figure 4.

In order to check that the function (6) is in fact a navigation function according to Definition 3.1, we still have to prove that it is a Morse function and smooth on $\mathcal{F}$. In this direction, first, we observe that the critical points of (6) are found by letting

$$\nabla_{q_i} V_i(q) = \frac{[k\beta_i(q)\nabla_{q_i} \Gamma_i(q) - \nabla_{q_i} \beta_i(q)]}{k(\Gamma_i(q)^k + \beta_i(q))^{1/k+1}} = 0,$$

(13)

that is

$$k\beta_i(q)\nabla_{q_i} \Gamma_i(q) = \Gamma_i(q)\nabla_{q_i} \beta_i(q).$$

(14)

On the other hand, this condition is valid in the following distinct cases:

(a) when $\nabla_{q_i} \Gamma_i(q) = 0$ (when $\Gamma_i(q) = 0$), $\nabla_{q_i} \beta_i(q) = 0$, and $\beta_i(q) = 1$: in this case, the identities $\nabla_{q_i} \beta_i(q) = 0$ and $\beta_i(q) = 1$ imply that the $i$-th vehicle is far from the boundary of the configuration space and other moving obstacles. In the meantime, the identity $\nabla_{q_i} \Gamma_i(q) = 0$ means that the vehicle is at the destination. We can

\[We will subsequently use this fact to infer that each UAV will converge to its goal point while avoiding obstacles.\]
prove that this equilibrium is in fact the minimum of the navigation function by evaluating the Hessian $\nabla^2_{\mathbf{q}} V_i(\mathbf{q})$ as shown in the following proposition.

**Proposition 3.2:** The navigation function (6) has a non-degenerate minimum when the $i$-th vehicle reaches its destination; this minimum is outside of all other vehicles’ protected zones.

**Proof.** See Appendix

(b) when $\nabla_{\mathbf{q}} \Gamma_i(\mathbf{q}) = 0$ (when $\Gamma_i(\mathbf{q}) = 0$), $\nabla_{\mathbf{q}} \beta_i(\mathbf{q}) = 0$, and $\beta_i(\mathbf{q}) = 0$: The first condition means the vehicle is at the destination. The identities $\nabla_{\mathbf{q}} \beta_i(\mathbf{q}) = 0$ and $\beta_i(\mathbf{q}) = 0$ also imply that the vehicle is at the boundary of an obstacle collision zone. This critical point is located at the boundary of the configuration space. However, Assumption 3.1 suggests that the vehicle’s destination is away from obstacles. Therefore, the vehicle can never be at this critical point.

(c) when $\nabla_{\mathbf{q}} \Gamma_i(\mathbf{q}) = 0$ (when $\Gamma_i(\mathbf{q}) = 0$) and $\nabla_{\mathbf{q}} \beta_i(\mathbf{q}) \neq 0$ (when $0 < \beta_i(\mathbf{q}) < 1$): The first condition means that the $i$-th vehicle is at the destination. The second condition translates to having the vehicle near other moving or static obstacles. Substituting these conditions into (33) yields

$$\nabla^2_{\mathbf{q}} V_i(\mathbf{q}) = \frac{\nabla^2_{\mathbf{q}} \beta_i(\mathbf{q})}{(\beta_i(\mathbf{q}))^{1/k}} = \frac{2I}{(\beta_i(\mathbf{q}))^{1/k}}$$

which implies that this point is non-degenerate. However, Assumption 3.1 suggests that the vehicle can never be at such a critical point.

(d) when $\nabla_{\mathbf{q}} \Gamma_i(\mathbf{q}) \neq 0$ (when $\Gamma_i(\mathbf{q}) \neq 0$) and $\nabla_{\mathbf{q}} \beta_i(\mathbf{q}) = 0$, and $\beta_i(\mathbf{q}) = 0$: The first condition means that the $i$-th vehicle is not at the destination. The identities $\nabla_{\mathbf{q}} \beta_i(\mathbf{q}) = 0$ and $\beta_i(\mathbf{q}) = 0$ also imply that the vehicle is at the boundary of an obstacle collision zone. Substituting these conditions into (33) yields

$$\nabla^2_{\mathbf{q}} V_i(\mathbf{q}) = -\frac{\nabla^2_{\mathbf{q}} \beta_i(\mathbf{q})}{k\Gamma_i'(\mathbf{q})}.$$  

Since we chose the bump function such that $f''(0) = 0$, it follows that $\nabla^2_{\mathbf{q}} \beta_i(\mathbf{q}) = 0$ at this point. However this type of critical point is not in the interior of the configuration space.

(e) when $\nabla_{\mathbf{q}} \Gamma_i(\mathbf{q}) \neq 0$ (when $\Gamma_i(\mathbf{q}) \neq 0$) and $\nabla_{\mathbf{q}} \beta_i(\mathbf{q}) \neq 0$ (when $0 < \beta_i(\mathbf{q}) < 1$), and $k\beta_i(\mathbf{q}) \nabla_{\mathbf{q}} \Gamma_i(\mathbf{q}) = \Gamma_i(\mathbf{q}) \nabla_{\mathbf{q}} \beta_i(\mathbf{q})$: The first condition means that the $i$-th vehicle is not at the destination; the second condition translates to having the vehicle near other moving obstacles. The equilibrium is not the desired one. In the meantime, the third condition leads to

$$k \frac{\nabla_{\mathbf{q}} \Gamma_i(\mathbf{q})}{\Gamma_i'(\mathbf{q})} = \frac{\nabla_{\mathbf{q}} \beta_i(\mathbf{q})}{\beta_i(\mathbf{q})},$$

which means that the gradients, due to the path planning and obstacle avoidance are in the same direction, but scaled by parameter $k$, and there exists a big enough $k$ that causes this undesired equilibrium to be pushed further toward the moving obstacle. The following proposition analyzes the stability of this type of critical points.

**Proposition 3.3:** The critical points analyzed above are in the interior of the free configuration space $\mathcal{F}$. There are at least as many saddle points as the number of obstacles. In fact, it is possible to find a lower bound for the parameter $k$ such that

1. each critical point of the navigation function is close to one of the obstacles, and
2. the critical points near the obstacles are not a local minimum, but only a saddle point, and not degenerate.

**Proof.** See Ref. [25].

From Propositions 3.2 and 3.3, we can conclude that (6) is a Morse function in $\mathcal{F}_i$.

**Proposition 3.4:** For each $\mathbf{q}_i$, the navigation function (6) is twice differentiable on $\mathcal{F}_i$.

**Proof.** From the expression for the Hessian of $V_i(\mathbf{q})$ in (33), we can see that $\Gamma_i(\mathbf{q})$ and $\beta_i(\mathbf{q})$ can not be zero at the same time in the interior of $\mathcal{F}_i$. Hence, the terms $\Gamma_i(\mathbf{q})^k + \beta_i(\mathbf{q}) > 0$ and $\nabla^2_{\mathbf{q}} V_i(\mathbf{q})$ are defined everywhere on $\mathcal{F}_i$.

While we show that at any instant of time, the function (6) satisfies all of the conditions in Definition 3.1, these conditions can only guarantee a goal point convergence, path attraction, and static obstacles avoidance, but not the collision avoidance with another (moving) UAV. This latter property will be guaranteed with the help of a swirling function presented in §IV. However, we first discuss two aspects of the navigation-based control law in the absence of swirling.

### A. Navigation-based UAV Control

The inner-loop controller of the UAVs to follow the desired velocity is of the form

$$u_i(t) = K_{\psi_i}(\psi^d_i(t) - \psi_i(t)) + \psi^d_i(t),$$

$$v_i(t) = K_{\gamma_i}(\gamma^d_i(t) - \gamma_i(t)) + \gamma^d_i(t),$$

where

$$\psi^d_i = \tan^{-1}_2 \left( v^d_{y_i}, v^d_{z_i} \right), \quad \gamma^d_i = \tan^{-1}_2 \left( v^d_{\psi_i}, \sqrt{(v^d_{x_i})^2 + (v^d_{y_i})^2} \right),$$

the superscript “d” indicates the desired value of the corresponding state, and $\tan^{-1}_2$ denotes the four quadrants arccotangent function. We now set $[v^d_{x_i}, v^d_{y_i}, v^d_{z_i}]^T = v^d_i(\mathbf{q}) = -\nabla_{\mathbf{q}} V_i(\mathbf{q})$, and $K_{\psi_i}$ and $K_{\gamma_i}$ are heading and pitching regulation gains, respectively. The feed-forward terms $\psi^d_i$ and $\gamma^d_i$ can now be calculated using $[\dot{v}^d_{\psi_i}, \dot{v}^d_{y_i}, \dot{v}^d_{z_i}]^T = \dot{v}^d_i(\mathbf{q}) = -\nabla_{\mathbf{q}} \nabla_{\mathbf{q}} V_i(\mathbf{q}) \dot{\mathbf{q}}$, which represents the rate of change of the potential’s slope as we travel in the $\mathbf{q}_i$ direction. These terms, in turn, increase the tracking performance of the UAVs to track the desired velocity $v^d_i(\mathbf{q})$ once the error has converged to zero. By solving (15) we have that

$$\psi_i(t) = \left| \psi_i(0) - \psi^d_i(0) \right| e^{-K_{\psi_i}t} + \psi^d_i(t)$$

and that implies that the error dynamics $\psi_i(t) - \psi^d_i(t) \to 0$ as $t \to \infty$.

Figure 5 depicts the gradient vector field through the configuration space and the parameters corresponding to UAV deconfliction including the swirling radius $r_{ss}$ that will be discussed in §IV. The gradient is numerically computed locally to save the computational cost. We note that the vector field varies in accordance with the location of the vehicle and its neighbor, and therefore it is a function of time and has to be recalculated at every time-step.
We incorporate all three way-points into constructing path-planning function $\Gamma(q)$. The location of the final goal point and the initial point are adjusted base on the the value of $y'$ in an auxiliary coordinate frame. These adjusted values are situated along the tangent of the curve during the transition. The point along the curve can be represented as $\tilde{q}_1 = [\sqrt{R^2_t - y'^2}; y'; 0]$ whereas the unit vector tangential to it is $\tilde{q}_2 = [-y'/R_t; \sqrt{R^2_t - y'^2}/R_t; 0]$. The adjusted goal point and the adjusted initial point can then be represented as

$$\begin{align*} 
\tilde{q}_3 &= \begin{cases} 
q_2 + \frac{|q_3 - q_2| - \|q_1 - q_2\| - R_t \tan \theta}{R_t \sin \theta} q_1, & y' \leq -R_t \sin \theta, \\
q_2 + \frac{|q_3 - q_2| - \|q_1 - q_2\| - R_t \tan \theta}{R_t \sin \theta} q_1, & |y'| < R_t \sin \theta, \\
q_2 - \frac{|q_2 - q_3| - \|q_2 - q_1\| - R_t \tan \theta}{R_t \sin \theta} q_1, & y' \geq R_t \sin \theta,
\end{cases} \\
\tilde{q}_3 &= \begin{cases} 
q_1 - \frac{|q_3 - q_2| - \|q_1 - q_2\| - R_t \tan \theta}{R_t \sin \theta} q_1, & y' \leq -R_t \sin \theta, \\
q_2 - \frac{|q_2 - q_3| - \|q_2 - q_1\| - R_t \tan \theta}{R_t \sin \theta} q_1, & |y'| < R_t \sin \theta, \\
q_2 - \frac{|q_2 - q_3| - \|q_2 - q_1\| - R_t \tan \theta}{R_t \sin \theta} q_1, & y' \geq R_t \sin \theta,
\end{cases}
\end{align*}$$

respectively. The adjusted path-planning function then becomes

$$\begin{align*} 
\tilde{\Gamma}(q) &= \|q - \tilde{q}_3\|^2 + K_p \beta \left( \frac{\|q - \tilde{q}_3\|}{R_p} \right) \tilde{d}^2,
\end{align*}$$

where $\tilde{d} = \|\tilde{q}_1 - \tilde{q}_3\| \times (q - \tilde{q}_3)/\|\tilde{q}_1 - \tilde{q}_3\|$. If desired, the obstacle function can also be included as suggested by (6). For the application where the way-point corners are small, this method requires the UAV to turn long before reaching the way-point. This may not be practical for a surveillance or monitoring mission. For a case like this, two way-points should be assigned accordingly to allow the UAV to make a turn surrounding the way-point of interest.

### IV. Swirling Function for UAV Deconfliction

Due to the nature of the unicycle model, necessitating control over heading only, and the fact that other UAVs are not “static obstacles,” the collision avoidance of the algorithm can not be guaranteed by merely following the negative gradient of the navigation function. Moreover, the navigation function can lead to an unbounded force or turn-rate commands which are not suitable for a unicycle model with turn-rate constraints. For example, if the protected zone radius is set too small as compared with the vehicle’s speed, the UAV needs to turn sharply in order to avoid the moving obstacle. For practical reasons, large turn-rates are neither desirable nor practical. In order to alleviate this situation, swirling effect has been introduced in Ref. [24] in order to improve the deconfliction performance and to help guarantee the collision avoidance under such dynamic limitations. The swirling effect also helps breaking the symmetry in order to avoid being trapped in saddle points of the navigation function around the moving obstacle.

A swirling effect is added to the gradient in order to “rotate” the aircraft counterclockwise around the nearby moving obstacle. This effect is augmented gradually from the distance when the deconfliction sensor detects the moving obstacle at distance $r_{ss}$; the effect is initialized with zero magnitude and is allowed to increase to the maximum level at the boundary.
of the collision zone. The parameter $K_s$ defines the maximum relative size of the swirling effect with respect to the original gradient. The spinning gradient vector field around the moving obstacles tends to break the symmetry for the case when two vehicles are exactly heading onto each other. For the case when the vehicles are ascending or descending along their respective desired paths, adjusting the heading command in the vertical direction can create higher a angle of attack which causes the aircraft to stall; therefore the swirling effect is added only along the horizontal direction while the desired vertical gradient at a corresponding location stays unaffected. The effect can be viewed as a vertical circular cylinder of a rotating vector field added to the original gradient of the navigation function as shown in Figure 7(c). The adjusted gradient for vehicle $i$ can be derived as,

$$\nabla V_{i}^{\text{new}} = \nabla V_i + K_s \beta \left( \frac{r_{ss} - ||q_i - q_j||}{r_{ss}} \right) \nabla V_i ||v_{i}^{ss}||,$$

$$v_{i}^{ss} = \frac{(q_i - q_j) \times k_z}{||q_i - q_j||},$$

(18)

where $q_i$ and $q_j$ are the locations of the UAV and the moving obstacle, respectively, and $k_z$ is the unit vector in the $z$ direction of the earth frame. The idea can also be extended for the case when more than one vehicle is within the range of detection, in this case the superposition of the effects from each vehicle may be used with a bump function (12) in order to emphasize the effects from the closer vehicles

$$\nabla V_{i}^{\text{new}} = \nabla V_i + \sum_{j \in N_i} K_{ij} \beta \left( \frac{r_{ss} - ||q_i - q_j||}{r_{ss}} \right) \nabla V_i ||v_{i}^{ss}||.$$  

Figure 5 shows the swirling effect around a moving obstacle with radius $r_{ss}$. Figures 7(a) and (b) compare the trajectories for the scenarios with and without the swirling effect, when two vehicles fly head-on into each other. For a completely symmetric case of this example, the original gradient direction will change to the opposite direction once the UAV enters the other UAV’s protected zone and creates a stagnation point, accompanied by a high turn-rate command. The counterclockwise swirling effect helps the UAV to always turn to the “right” when encountering an obstacle. However this effect creates a trajectory overshoot once the collision is resolved as the swirling component still continues to rotate the aircraft after it passes the moving vehicle; it might also steer the aircraft into a high turn rate zone. We thereby propose an overshoot alleviation mechanism by reducing the swirling effect following the resolution of the conflict.

A. Trajectories Overshoot and Swirling Effects Reduction

The overshoot caused by the swirling effect reduces the aircraft tracking performance and results in some delay for the aircraft to go back to the nominal path. One of the ways to solve this problem is to reduce the swirling effect once two vehicles are out of the course of collision. For this purpose, we employ the concept of collision cone and relative velocity used in Refs. [7], [10], [13]. A collision cone is defined by the area inside the lines drawn from the vehicle to the obstacle’s collision zone. The two vehicles are out of the course of collision if their relative velocity stays outside the collision cone. Next, let us define the safety angle to help measure how far the two vehicles are from the collision.

**Definition 4.1:** A safety angle of a collision for vehicle $i$ from vehicle $j$, $\theta_{ss}$, is the angle between the relative velocity $v_{ij} = v_i - v_j$ and the collision cone drawn from vehicle $i$ to vehicle $j$’s collision zone, where $v_i$ indicates the velocity vector of vehicle $i$ (this is shown in Figure 8(a)).

As this relative velocity stays farther from the collision cone (larger safety angle), the two vehicles are “less in conflict.” Our approach is to reduce the swirling radius according to the safety angle from $r_{ss}$ at the cut-off angle $\theta_{\text{cutoff}}$ to zero when the safety angle equals 180 degree. The swirling reduction is done by the same pattern using the bump function in (12). For smooth transition of the aircraft’s trajectory, the cutoff angle is chosen to be a value in the interval of $[\pi/4, 3\pi/8]$ as shown in Figure 8(b). The swirling radius is then adjusted as

$$r_{ss}^{\text{adjusted}}(\theta_s) = \left\{ \begin{array}{ll} r_{ss} & \theta_s \leq \theta_{\text{cutoff}}; \\ r_{ss} \frac{\theta_s - \theta_{\text{cutoff}}}{\pi/8 - \theta_{\text{cutoff}}} & \theta_s > \theta_{\text{cutoff}}. \end{array} \right.$$  

(20)

For the case when more than 2 vehicles are in the range of detection, the swirling effect from each neighbor are summed according to (19) and the radius $r_{ss}$, or the relative magnitude $K_s$, is adjusted based on the safety angle $\theta_{ss}$ between each pair of vehicles. Figure 8(c) shows that this approach helps prevent the vehicle from having an overshoot into the high turn-rate zone, a topic that will be discussed further in §IV-C.

B. Swirling Effect and Guaranteed Deconfliction

In this section, we explore the guaranteed convergence of the deconfliction algorithm to collision-free state under the navigation law dictated by the swirling effect. In particular,
we prove that when two vehicles are within the radar range of each other (distance $r_{ss}$ apart), their protected zones will not intersect at all times until they exit the proximity detection zone. In order to simplify the proof, we will adopt the following two assumptions, the second of which will be subsequently relaxed:

Assumption 4.1: Since the swirling effect is added along the horizontal direction while the vertical desired heading derived from the navigation function remains unchanged, the guaranteed deconfliction can be viewed from the projection of the flight paths onto the horizontal plane. It is thus sufficient to show that the swirling effect keeps the two vehicles apart if it achieves this objective on the projected horizontal plane. Therefore, we can examine the guaranteed deconfliction for the case of two vehicles on the same plane and adopt the model,

$$
\begin{align*}
\dot{x}_i(t) &= U_h \cos \psi_i(t), \\
\dot{y}_i(t) &= U_h \sin \psi_i(t), \\
\dot{\psi}_i(t) &= K \psi_i (\psi^d_i(t) - \psi_i(t)) + \psi^d_i(t),
\end{align*}
$$

(21)

where $\psi^d_i$ $(i=1, 2)$ is the desired headings for the UAV $i$ as derived from (16) and (18) and $U_h$ is the projection of its velocity to the horizontal plane.

Assumption 4.2: For simplification of the proof, we first look at the case where two conflicting UAVs has the same flight path angle or $\cos \gamma_1 = \cos \gamma_2$ and that $U_{h_1} = U_{h_2} = U_h$. Since we only look at projection in 2D, the subscript $h$ is dropped in the notation. The result when $U_{h_1} \neq U_{h_2}$ will be discussed toward the end of the section.

Assumption 4.3: During the deconflicting maneuver, the vehicles’ aim to avoid entering each other protected zones; hence the gradient due to the navigation function comes from the path planning term only. The original gradient (before adding swirling term) is rather uniform around the area that is far from the destination. By neglecting this term with respect to the swirling effect, the behavior of deconflicting maneuver is expected to be minimally effected. Thus, we shall first drop the contribution of the navigation function in our collision avoidance proof below; in §IV-C , we provide a collision avoidance analysis when this term is added back. In this former case (18) assumes the form

$$
\nabla V_i = \frac{(q_i - q_j) \times \vec{k}_z}{\|q_i - q_j\|},
$$

and accordingly the desired heading is

$$
\psi^d_i(t) \approx \psi^{d,ss}_i(t) = \text{atan}_2(x_i(t) - x_j(t), y_j(t) - y_i(t)),
$$

(22)

where the superscript "ss" denotes the desired heading obtained from the swirling effect. Hence, the feed-forward term can be derived from

$$
\dot{\psi}^{d,ss}_i(t) = \frac{\cos \psi^{d,i}(t) - \cos \psi^d_i(t) + \sin \psi^{d,i}(t) \sin \psi^d_i(t)}{\sqrt{(x_i(t) - x_j(t))^2 + (y_j(t) - y_i(t))^2}}.
$$

Now by letting $\bar{x}(t) = x_1(t) - x_2(t), \bar{y}(t) = y_1(t) - y_2(t)$, the system of two vehicles under the swirling effects can be represented as,

$$
\begin{align*}
\dot{\bar{x}}(t) &= U \cos \psi_1(t) - \cos \psi_2(t), \\
\dot{\bar{y}}(t) &= U \sin \psi_1(t) - \sin \psi_2(t), \\
\dot{\psi}_1(t) &= K (\psi^d_1(t) - \psi_1(t)) + \psi^d_1(t), \\
\dot{\psi}_2(t) &= K (\psi^d_2(t) - \psi_2(t)) + \psi^d_2(t),
\end{align*}
$$

(23)

with

$$
\psi^{d,ss}_1(t) = \text{atan}_2(\bar{x}(t), -\bar{y}(t)), \psi^{d,ss}_2(t) = \text{atan}_2(-\bar{x}(t), \bar{y}(t)),
$$

where $K$ represents the inner-loop control gain of both vehicles $K_{\psi_1}$ and $K_{\psi_2}$; additionally, $\psi_{10}$ and $\psi_{20}$ are the initial headings when the UAVs enter the swirling zone at distance $r_{ss}$ apart.

Figure 9 depicts the trajectories of two vehicles according to (23) with 4 different pairs of initial headings. Note that the swirling effect drives the two vehicles to converge into the same circular limit cycle and have them stay on the opposite side of the circle heading in opposite directions. The headings at the equilibrium are equal to the desired headings, i.e., $\psi_i = \psi^{d,ss}_i$. In the meantime, different pairs of initial heading results in convergence to different limit cycles.

In order to prove the behavior of the pair of UAVs under the swirling effect, we use LaSalle’s Invariance Principle to show that the vehicles in (23) converge to a circular limit cycle, whose diameter is the minimum distance between the two UAVs. In fact, this limit cycle can be written as an invariant set

$$
\mathcal{M} = \{(\bar{x}, \bar{y}, \psi_1, \psi_2) \in \mathbb{R}^4; \bar{x}^2 + \bar{y}^2 = R^2, \psi_1 = \text{atan}_2(\bar{x}, -\bar{y}), \psi_2 = \text{atan}_2(-\bar{x}, \bar{y})\},
$$

(24)

where $R$ is a radius of the limit cycle.

Theorem 4.2: The system of two UAVs under the swirling effect in (23) will converge to the limit cycle (24) and will keep a distance greater than $2R$ when initialized from $r_{ss}$ apart with $\psi_{10}$ and $\psi_{20}$ initial headings.
Proof. First, we show that $\mathcal{M}$ is positively invariant by

$$
\frac{d}{dt}(\bar{x}^2 + \bar{y}^2 - R^2) = 2(\bar{x}\ddot{x} + \bar{y}\ddot{y})
= 2U(\bar{x}(\cos \psi_1 - \cos \psi_2) + \bar{y}(\sin \psi_1 - \sin \psi_2))
= 4UR\sin(\psi_1^{d,ss} - \psi_1) + \sin(\psi_2^{d,ss} - \psi_2)) = 0
$$

and hence, if $(\bar{x}(0), \bar{y}(0), \psi_1(0), \psi_2(0)) \in \mathcal{M}$, then $(\bar{x}(t), \bar{y}(t), \psi_1(t), \psi_2(t)) \in \mathcal{M}$, for $t \geq 0$. The motion in the set $\mathcal{M}$ is characterized by $\psi_1(t) = \psi_1(0) = U/R$ which shows that $\mathcal{M}$ is a limit cycle for $(23)$, where the state vector moves counterclockwise. To examine whether $\mathcal{M}$ is attractive, consider the continuously differentiable function $V : \mathbb{R}^4 \to \mathbb{R}$ representing the deviation of the system from being on the limit cycle as

$$
V(\bar{x}, \bar{y}, \psi_1, \psi_2) = (\bar{x}^2 + \bar{y}^2 - R^2)^2 + (\psi_1^{d,ss} - \psi_1)^2 + (\psi_2^{d,ss} - \psi_2)^2
$$

and consider its time derivative

$$
\dot{V}(\bar{x}, \bar{y}, \psi_1, \psi_2) = 4(\bar{x}^2 + \bar{y}^2 - R^2)(\ddot{x} + \ddot{y})
+ 2(\psi_1^{d,ss} - \psi_1)(\dot{\psi}_1^{d,ss} - \dot{\psi}_1) + 2(\psi_2^{d,ss} - \psi_2)(\dot{\psi}_2^{d,ss} - \dot{\psi}_2).
$$

It is straightforward to see that when the trajectories start on the limit cycle $\mathcal{M}$ or $(\bar{x}(0), \bar{y}(0), \psi_1(0), \psi_2(0)) = (r_{ss}, 0, \psi_1^{d,ss}(0), \psi_2^{d,ss}(0))$, they will stay on $\mathcal{M}$ for $t \geq 0$ and $\bar{x}^2 + \bar{y}^2 = R^2$, $\psi_1 = \psi_1^{d,ss}$, $\psi_2 = \psi_2^{d,ss}$. Hence in this case $V = 0$.

We now proceed to examine the case when the two vehicles start outside the limit cycle. In this venue, let

$$
\dot{V}_1 = 4(\bar{x}^2 + \bar{y}^2 - R^2)(\ddot{x} + \ddot{y})
= 4\dot{U}(\bar{x}\cos \psi_1 - \cos \psi_2) + \dot{\bar{y}}(\sin \psi_1 - \sin \psi_2)
= 4U(\bar{x}\ddot{x} + \ddot{y})(\bar{x}\sin(\psi_1^{d,ss} - \psi_1)) + \sin(\psi_2^{d,ss} - \psi_2).
$$

We now check the inequality $\dot{V}_1 < 0$ by examining the following distinct cases:

(a) Two vehicles start with a course of collision: As $\psi_{10}, \psi_{20}$ face each other (or $\psi_{10} \in (-\frac{3\pi}{2}, -\frac{\pi}{2})$, $\psi_{20} \in (\frac{\pi}{2}, \frac{3\pi}{2})$), swirling effects will direct the vehicles by $\psi_1^{d,ss}, \psi_2^{d,ss}$ to go around each other counterclockwise and converge to $\mathcal{M}$. Both vehicles converge to $\mathcal{M}$ from outside and that $\bar{x}^2 + \bar{y}^2 > R^2$. Figure 10(a) shows that $-\pi < \psi_1^{d,ss} - \psi_1 < 0$, $-\pi < \psi_2^{d,ss} - \psi_2 < 0$ for all time before the trajectories approach $\mathcal{M}$ and $\sin(\psi_1^{d,ss} - \psi_1) < 0, \sin(\psi_2^{d,ss} - \psi_2) < 0$. Hence, $\dot{V}_1 < 0$.

(b) Two vehicles start facing away from each other: As $\psi_{10}, \psi_{20}$ face away from each other (or $\psi_{10} \in (\frac{\pi}{2}, \frac{3\pi}{2})$, $\psi_{20} \in (-\frac{3\pi}{2}, -\frac{\pi}{2})$), both vehicles converge to $\mathcal{M}$ from inside and that $\bar{x}^2 + \bar{y}^2 < R^2$. Figure 10(b) shows that $0 < \psi_1^{d,ss} - \psi_1 < \pi$, $0 < \psi_2^{d,ss} - \psi_2 < \pi$ for all time before the trajectories approach $\mathcal{M}$ and $\sin(\psi_1^{d,ss} - \psi_1) > 0, \sin(\psi_2^{d,ss} - \psi_2) > 0$. Hence, $\dot{V}_1 < 0$.

(c) Two vehicles start with a similar direction: without loss of generality, we consider the case where $\psi_{10} \in (-\frac{3\pi}{2}, -\frac{\pi}{2})$, $\psi_{20} \in (-\frac{3\pi}{2}, -\frac{\pi}{2})$ (Figure 10(c)) and that $\sin(\psi_1^{d,ss} - \psi_1) > 0$ but $\sin(\psi_2^{d,ss} - \psi_2) > 0$. We notice that if $|\sin(\psi_1^{d,ss} - \psi_1)| > |\sin(\psi_2^{d,ss} - \psi_2)|$, it requires more control effort for aircraft one to converge to $\mathcal{M}$ and the trajectory in $\bar{x}\bar{y}$-plane converges to $\mathcal{M}$ from outside and that $\bar{x}^2 + \bar{y}^2 > R^2$. Hence, $\dot{V}_1 < 0$. The case when $|\sin(\psi_1^{d,ss} - \psi_1)| < |\sin(\psi_2^{d,ss} - \psi_2)|$, the trajectory in $\bar{x}\bar{y}$-plane converges to $\mathcal{M}$ from inside. Since $\bar{x}^2 + \bar{y}^2 < R^2$ in this case, the inequality $\dot{V}_1 < 0$ also holds.

It is noted that in practice, swirling effects are not applied for cases (b) and (c), since the vehicles will move farther from $r_{ss}$ from each other and the safety angle will be $\frac{\pi}{4}$ off from the corresponding collision cone.

We now observe that

$$
\dot{V}_2 = 2(\psi_1^{d,ss} - \psi_1)(\dot{\psi}_1^{d,ss} - \dot{\psi}_1) + 2(\dot{\psi}_2^{d,ss} - \dot{\psi}_2)(\psi_2^{d,ss} - \psi_2)
= 2(\psi_1^{d,ss} - \psi_1)(\dot{\psi}_1^{d,ss} - \dot{\psi}_1 - K(\psi_1^{d,ss} - \psi_1))
+ 2(\dot{\psi}_2^{d,ss} - \dot{\psi}_2)(\psi_2^{d,ss} - \psi_2 - K(\psi_2^{d,ss} - \psi_2))

= -2K(\psi_1^{d,ss} - \psi_1^2 + (\psi_2^{d,ss} - \psi_2)^2) < 0,
$$

which translates to having $\dot{V} < 0$ anywhere outside $\mathcal{M}$. Next, let $\beta > 0$ and define

$$
\mathcal{D}_c = \{ (\bar{x}, \bar{y}, \psi_1, \psi_2) \in \mathbb{R}^4; V(\bar{x}, \bar{y}, \psi_1, \psi_2) \leq \beta \}.
$$

We note that $\dot{V} \leq 0$ everywhere in $\mathcal{D}_c$. Next, defining $\mathcal{R} = \{ (\bar{x}, \bar{y}, \psi_1, \psi_2) \in \mathbb{R}^4; V(\bar{x}, \bar{y}, \psi_1, \psi_2) = 0 \}$; it thus follows that the largest invariant set in $\mathcal{R}$ is $\mathcal{M}$. Hence, the system of two vehicles, under swirling effect in $(23)$, with $(\bar{x}(0), \bar{y}(0), \psi_1(0), \psi_2(0)) = (-r_{ss}, 0, \psi_{10}, \psi_{20}) \subseteq \mathcal{D}_c$, one has $(\bar{x}(t), \bar{y}(t), \psi_1(t), \psi_2(t)) \to \mathcal{M}$ as $t \to \infty$. ■

Theorem 4.2 shows that two vehicles converge to a limit cycle assuming that the gradient from the goal and path attraction is negligible as compared with that of the swirling effect. The effect of the neglected terms are analyzed in §IV-C. Moreover, when the swirling effect reduction method is applied, both UAVs will converge and stay on the limit cycle only until they point away from each other, at which point they will continue on their original paths. Our result formalizes the fact the swirling effect keeps the vehicles apart by the diameter of the limit cycle $\mathcal{M}$.

It is also interesting to see the relationship between designed parameters and the minimum distance between two aircraft. In fact, the distance between the two vehicles in $(23)$ is $d(t) = \sqrt{\bar{x}(t)^2 + \bar{y}(t)^2}$ and its time derivative can be found as

$$
d(t) = \frac{1}{d(t)}(\bar{x}(t)\dot{x}(t) + \bar{y}(t)\dot{y}(t))
= U\left(\bar{x}(t)(\cos \psi_1(t) - \cos \psi_2(t)) + \bar{y}(t)(\sin \psi_1(t) - \sin \psi_2(t))\right)
= U(\sin(\psi_1^{d,ss}(t) - \psi_1(t)) + \sin(\psi_2^{d,ss}(t) - \psi_2(t))).
$$
For \( i = 1, 2 \), let \( \alpha_i(t) = \psi_i^{d,ss}(t) - \psi_i(t); \) then
\[
\dot{\alpha}_i(t) = \psi_i^{d,ss}(t) - [K(\psi_i^{d,ss}(t) - \psi_i(t)) + \psi_i^{d,ss}(t)] \\
= -K\alpha_i(t). \tag{25}
\]

The distance between two vehicles under swirling effect in (23) is thereby governed by
\[
d(t) = U|\sin\alpha_1(t) + \sin\alpha_2(t)|, \quad d(0) = r_{ss} \tag{26}
\]
\[
\dot{\alpha}_1(t) = -K\alpha_1(t), \quad \alpha_1(0) = \alpha_{i0} \in [-\frac{\pi}{2}, -\theta_{\text{cutoff}}, 0] \\
\dot{\alpha}_2(t) = -K\alpha_2(t), \quad \alpha_2(0) = \alpha_{i0} \in [-\frac{\pi}{2}, -\theta_{\text{cutoff}}, 0]
\]

where the initial headings \( \alpha_{i0}, \alpha_{j0} \in [-\frac{\pi}{2}, -\theta_{\text{cutoff}}, 0] \) are only for case (a). The minimum distance between two vehicles can then be derived by solving (26).

**Theorem 4.3:** The system of two vehicles under swirling effects (23) has the minimum distance between them as
\[
d_{\text{min}} = r_{ss} - \frac{U}{K}[\text{Si}(\alpha_{i0}) + \text{Si}(\alpha_{j0})],
\]
where \( \text{Si}(x) \) is a sine integral function defined as \[26\]
\[
\int_0^x \sin t \, dt.
\]

**Proof.** From (26), we have \( \alpha_1(t) = \alpha_{i0}e^{-Kt} \) and \( \alpha_2(t) = \alpha_{j0}e^{-Kt} \) and the distance between two vehicles is thus
\[
d(t) = d(0) + \int_0^t U|\sin\alpha_1(t) + \sin\alpha_2(t)| \, dt \\
= r_{ss} - \frac{U}{K}\int_{\alpha_{i0}}^{\alpha_{j0}} \sin \frac{\alpha_1}{\alpha_1} d\alpha_1 + \int_{\alpha_{i0}}^{\alpha_{j0}} \sin \frac{\alpha_2}{\alpha_2} d\alpha_2 \\
= r_{ss} - \frac{U}{K}[\text{Si}(\alpha_{i0}) - \text{Si}(\alpha_{j0})] \\
+ \frac{U}{K}[\text{Si}(\alpha_{i0}) - \text{Si}(\alpha_{j0})].
\]

The minimum distance is hence found from
\[
d_{\text{min}} = \lim_{t \to \infty} d(t) = r_{ss} - \frac{U}{K}[\text{Si}(\alpha_{i0}) + \text{Si}(\alpha_{j0})]. \tag{27}
\]

The worst initial headings that lead to the shortest minimum distance occur when the \( \text{Si} \)-function reaches its maximum at \( \alpha_{i0} = \alpha_{j0} = -\pi \) (This is not for the case when a “safety angle” has been used). The minimum distance therefore becomes
\[
d_{\text{min}} = r_{ss} - \frac{2U}{K}[\text{Si}(\pi)] \approx r_{ss} - \frac{3.704U}{K}.
\]

The formula (27) shows the relationship between aircraft cruising speed \( U \), control gain \( K \), the effective range of detection \( r_{ss} \), initial headings, as well as the minimum distance between the two vehicles. In tuning of the parameters for the deconfliction algorithm, we thus aim to have the minimum distance be greater than the radius of vehicles’ protected zones.

For the case when \( U_{h1} \neq U_{h2} \), the UAVs converge to two different limit cycles that share the same center but different in radius and stay on the opposite side of the center heading in opposite direction. The limit cycle can be written similar to (24) in the space of \((\hat{x}, \hat{y}, \psi_1, \psi_2)\). The minimum distance can also be found from \( d_{\text{min}} = r_{ss} - \frac{U_{h1}}{K_1}\text{Si}(\alpha_{i0}) - \frac{U_{h2}}{K_2}\text{Si}(\alpha_{j0}) \).

### C. Guaranteed Deconfliction for Combined Navigation and Swirling Functions

In this section, we explore the minimum distance between a pair of conflicting UAVs when the navigation-based trajectory generation, swirling effect, and swirling effect reduction, are combined. In this venue, we continue to adopt Assumption 4.1 and Assumption 4.2 but drop Assumption 4.3 to see the effects of the path planning gradient on the minimum distance between the vehicles. First, we restate (18) in the polar coordinates as
\[
||\nabla_{\psi_i}^\text{naw}||\psi_i = 1\psi_i^d,\text{path} + K_3\beta \left( \frac{r_{ss} - ||q_i - q_0||}{r_{ss}} \right) \psi_i^d,\text{ss}, \tag{28}
\]
where \( \psi_i^d, \psi_i^d, \psi_i^d \) are the desired heading, the desired heading from the path planning, and the desired heading from the swirling effect, respectively. The following assumption is introduced in order to streamline the subsequent proof.

**Assumption 4.4:** At a location far from the destination, \( \psi_i^d,\text{path} \) is uniform and pointed toward the goal. Rather than neglecting this term, we invoke that during the avoidance maneuver, the path planning gradient points in the same direction as when the vehicle enters the swirling zone, i.e., that \( \psi_i^d,\text{path} \approx \psi_{i0} \).

Since \( \psi_i^d \neq \psi_i^d,\text{ss} \), for \( i = 1, 2 \), we define \( \alpha_i(t) = \psi_i^d(t) - \psi_{i0} \) and we have that \( \dot{\alpha}_i(t) = -K\alpha_i(t) \). When \( t = 0 \) at the beginning of the swirling zone where \( \|q_1 - q_2\| = r_{ss} \), \( \beta(0) = 0 \), \( \alpha_1(0) = \alpha_2(0) = 0 \) which translates to having \( \dot{\alpha}_1(t) = \dot{\alpha}_2(t) = 0 \) for all \( t > 0 \). This shows that \( \psi_i^d(t) = \psi_{i0} \) as long as \( \psi_i^d(t) \) is smooth and the feedback gain \( K \) is large enough to track \( \psi_{i0} \). The minimum distance between two UAVs can now be derived from as
\[
d_{\text{min}} = d(0) + \int_0^{t_{\text{min}}} \left| U\sin(\psi_1^{d,ss}(t) - \psi_1^d(t)) + \sin(\psi_2^{d,ss}(t) - \psi_2^d(t)) \right| dt, \tag{29}
\]
where \( t_{\text{min}} \) is the time when the distance between two vehicles reaches its minimum value. The following lemma describes the main feature of the function \( \psi_i^{d,ss}(t) - \psi_i^d(t), i = 1, 2 \).

**Lemma 4.4:** For \( i = 1, 2 \), the function \( |\alpha_i(t)| = |\psi_i^{d,ss}(t) - \psi_i^d(t)| \) is a decreasing function of time on the interval \( (0, t_{\text{min}}) \), where \( \alpha_i(0) = \alpha_{i0} \in [-\frac{\pi}{2}, -\theta_{\text{cutoff}}, 0] \), \( \alpha_i(t_{\text{min}}) = 0 \) and \( \frac{d}{dt} |\psi_i^{d,ss}(t) - \psi_i^d(t)|_{t=t_{\text{min}}} = 0 \).

**Proof.** From (28), we have that \( \psi_i^d,\text{path} = \psi_{i0} \) and the term \( \beta \left( \frac{r_{ss} - ||q_i - q_0||}{r_{ss}} \right) \) grows away zero towards the value of one as the two vehicles come closer to each other, which monotonically decreases \( |\alpha_i(t)| \) until the minimum distance between the two UAVs is reached. In addition to this, \( \psi_i^{d,ss} \) also heads toward \( \psi_{i0} \) as the swirling effect progresses until \( |\alpha_i(t)| = |\psi_i^{d,ss}(t) - \psi_i^d(t)| = 0 \) at \( t = t_{\text{min}} \). After reaching the minimum distance, the value of \( \psi_i^{d,ss} \) grows on the opposite side from \( \psi_i^d \) to push the vehicle back to the nominal path heading \( \psi_{i0} \). The plot in Figure 11(a) shows \( \psi_i^{d,ss}(t) - \psi_i^d(t), i = 1, 2 \) for the head-on case. Furthermore, the desired heading can also be written of the form \( \psi_i^d = \psi_{i0} + atan_2 \left( K_2\beta \sin(\psi_i^{d,ss}(t) - \psi_{i0}), 1 + K_2\beta \cos(\psi_i^{d,ss}(t) - \psi_{i0}) \right) \) and by letting \( t \to 0 \), \( \beta(0) = 0 \), and \( \beta'(0) = 0 \), the time derivative of the desired heading becomes \( \frac{d}{dt} |\psi_i^{d,ss}(t) - \psi_i^d(t)|_{t=0} = \frac{d}{dt} |\psi_i^{d,ss}(t) - \psi_i^d(t)|_{t=0} = 0 \).
integrating (29) numerically from \( t = 0 \) to \( t_{\min} \), where \( \psi_i^{d,ss}(t) - \psi_i^{d}(t) = 0 \), determining the value \( d_{\min} \). This process can then be iterated up with reduced values of \( K_s \) until the resulting value for \( d_{\min} \) reaches the radius of the protected zones for the UAVs.

When the proposed deconfliction algorithm is applied to \( n \) vehicle scenarios with \( n > 2 \), in most cases, the intersection of regions of detection occurs between a pair of vehicles at a time and the conflict is resolved before detecting another vehicle. Hence the swirling effect is only applied to the closest UAV and (30) can guarantee the minimum distance between the two deconflicting vehicles. For the case when other vehicles have overlapping detection zones with a UAV at exactly the same instance, or before the swirling effect between two vehicles vanishes, the superposition of the effects from each vehicles can be used with bump function in order to emphasize the effect from the closer vehicles as shown in (19). The construction of Lyapunov-like function to prove the convergence for this more general case turns out to be more involved as the number of variables increases and the corresponding limit cycles also vary with the different conflicting scenarios. For example, when \( n \) vehicles symmetrically come into conflict from \( \frac{2\pi}{n} \) degrees apart and (19) is applied (along with Assumption 4.3), the vehicles will converge to a circular limit cycle and stay equidistant around the circle with \( \frac{2\pi}{n} \) phase different, and move counterclockwise until each of them reaches its nominal path. For the case when \( n \) vehicles come into conflict but not in a symmetrical way, the limit cycle is no longer unique and each vehicle will have its own periodic limit cycle. However, the theoretical guarantees for the general \( n \) unicycle-type UAVs with turn-rate constraints is not established in the present work and is the subject of future work.

V. UAV DECONFlictION UNDER TURN-RATES CONSTRAINTS

For better tracking performance, the algorithm should generate commands that are within the aircraft acceleration constraints (2). In order to limit the turn-rate commands during the deconfliction maneuver, we proceed to examine the spatial derivatives of the gradient vector across the free configuration space. Since the magnitude of the gradient is not used in our approach, we only consider the change of its direction. The goal is to create a navigation function and swirling effect with derivatives that fall within the turn rate limit.

Intuitively the gradient should change with a lower rate when the navigation function gets smoother and less steep around the moving obstacle. One way to achieve this is to increase the protected zone radius as shown in Figure 12. However, the swirling radius is modeled based on how early generated by the purely swirling effects base on \( K_s \), one can embed a safety factor into (27) by substituting \( K \) with \( K_r \), where \( 0 < K < K_r \). For smooth trajectories, one can choose \( K_r = 0.7K \).

2) Based on the chosen value of \( r_{ss} \), the on-board computer for the UAVs can determine the value of \( K_s \) by first setting it comparatively high, (e.g., \( K_s = 10 \)) and integrating (29) numerically from \( t = 0 \) to \( t_{\min} \), where \( \psi_i^{d,ss}(t) - \psi_i^{d}(t) = 0 \), determining the value \( d_{\min} \). This process can then be iterated up with reduced values of \( K_s \) until the resulting value for \( d_{\min} \) reaches the radius of the protected zones for the UAVs.

We can now use the result from Lemma 4.4 to obtain a bound on the minimum distance in (29) when the trajectories of the UAV pair are generated via the combined navigation and swirling functions.

Theorem 4.5: For a pair of UAVs (21) for which the desired headings \( \psi_i^d \), \( i = 1, 2 \) are derived from navigation-based path-planning and swirling effect in (28), there exists a constant \( K < K_s \), such that the minimum distance between the UAVs is guaranteed to be bounded by \( r_{ss} - \frac{2U}{K_s}[\sin(\pi) - \sin(\pi e^{-Kt_{\min}})] \) during their entire trajectories.

Proof. From §IV-A, let us choose \( \delta_{\text{offset}} \) to be in \( \left[ \frac{2\pi}{n}, \frac{3\pi}{n} \right] \) which implies that \( \alpha_i(0) \in \left[ \frac{-2\pi}{K_s}, 0 \right] \). Lemma 4.4 also shows that \( \alpha_i(t) \) is a decreasing function on the time interval \( (0, t_{\min}) \), \( \alpha_i(0) = 0 \) and \( \frac{d}{dt} \alpha_i(t) \mid_{t=0} = 0 \). Therefore, we can always find \( \tilde{K}, 0 < \tilde{K} \leq K_s \), such that \( -\pi < \pi e^{-\tilde{K}t} < \alpha_i(t) < 0 \) for all \( t \in (0, t_{\min}) \). Furthermore, \( \sin(\pi e^{-\tilde{K}t}) < 0 \) for all \( t \in (0, t_{\min}) \), and thus the minimum distance is

\[

\begin{align*}
    d_{\min} &= r_{ss} + \int_0^{t_{\min}} U[\sin(\alpha_i(t)) + \sin(\alpha_2(t))] \, dt \\
    &> r_{ss} + \int_0^{t_{\min}} 2U[\sin(\pi e^{-\tilde{K}t})] \, dt \\
    &= r_{ss} - \frac{2U}{K_s}[\sin(\pi) - \sin(\pi e^{-\tilde{K}t_{\min}})] \, .
\end{align*}

\]

For the design purpose, we would like determine how the swirling parameter \( K_s \) affects the value of \( \tilde{K} \) and \( t_{\min} \) which would in turn, determine the minimum distance between the two vehicles. The plot in Figure 11(b) shows that the larger values for \( K_s \) leads the function \( \alpha_i(t) \) to behave similar to the case of purely swirling in Theorem 4.3, that is \( \alpha_i(t) \) behaves similar to \( \alpha_i e^{-Kt} \) as \( K_s \) increases. This suggests that the more emphasis of swirling effect results in the bigger value of \( \tilde{K} \) and \( t_{\min} \) and hence a larger bound on the minimum distance. However, the relationship between \( K_s \) and \( K \) is nonlinear and in practice, the value of \( K_s \) can be found by numerically integrating (29).

The selection of the parameters in the combined navigation and swirling functions-based trajectory generation algorithm can now be presented as follows:

1) Given the required minimum distance \( d_{\min} \), the cruising speed \( U \), the control gain \( K \), and the initial heading \( \alpha_{i_1}, \alpha_{i_2} \), for the pair of UAVs, one can use (27) to approximate \( r_{ss} \) or how early the swirling effect should take place. Since (27) is made under Assumption 4.3 and the actual trajectory “rotates” slower than the one
that ∇ based on different values of k. Figure 4 shows that the potential level can swing up and down the emphasis between obstacle avoidance and goal attraction.

A. Tuning Parameter k

As mentioned in the previous section, the parameter k scales the emphasis between obstacle avoidance and goal attraction. Figure 4 shows that the potential level can swing up and down based on different values of k. However, as shown below, the value of k does not affect the aircraft turn rate.

Proposition 5.1: The tuning parameter k of the potential function in (6) does not change the direction of its gradient if the vehicle stays outside other vehicle’s protected zone.

Proof. From (13), the gradient of the navigation function is

\[ \nabla q_i V_i(q) = \frac{k \beta_i(q) \nabla q_i \Gamma_i(q) - \Gamma_i(q) \nabla q_i \beta_i(q)}{k(\Gamma_i(q) k + \beta_i(q))^{1/k+1}}. \]

While the aircraft stays outside other vehicles’ protected zones, \( \beta(q) \) stays constant at one and \( \nabla q_i \beta_i = 0 \). Thus the gradient becomes \( \nabla q_i V_i(q) = (\Gamma_i^k + 1)^{-1/k+1} \nabla q_i \Gamma_i(q) \Gamma_i(q) \Gamma_i(q) \) and (7) shows that \( \nabla q_i \Gamma(q) \) is not a function of k. Hence, k can only change the magnitude, but not the direction, of the gradient, and hence, the turn-rate is independent of the value of k.

B. The Path Attraction Parameters \( K_p \) and \( r_p \)

Increasing the parameter \( K_p \) provides a scale for emphasizing the path attraction and higher turn rate when the vehicle swings back to the nominal path to head toward its goal. Figure 13 demonstrates how the prescribed trajectory changes with different values of \( K_p \). This effect is dimmed down when the vehicle gets closer to the goal within the distance \( r_p \), thus the turn rate varies conversely with \( r_p \). These parameters have more pronounced effect on the vehicle trajectory when the aircraft turns into its nominal path and not when they turn to avoid other vehicles. The following proposition provides the analytic formula that assists in choosing \( K_p \) and \( r_p \) such that the path attraction turn rate does not exceed the UAV dynamic constraints.

Proposition 5.2: When an aircraft (1) with a unity constant speed and the deconfliction algorithm (5), (6), (7), and (8), turn to the planned-path toward the goal point, the path attraction turn rate is bounded by \( K_p \sqrt{2} \). \( \triangle \)

Proof. We first note that when the vehicle stays off its nominal path by d at the distance \( L \geq r_p \) from the goal, the term \( \beta_i(\|q_i - q_d\| / r_p) = 1 \) and (7) becomes

\[ \Gamma = (L^2 + d^2) + K_p d^2 = L^2 + (1 + K_p) d^2. \]

Since we consider only the position outside other vehicle’s protected zone, the function \( \beta \) in (6) remains constant at one. The angle \( \theta \) between the gradient vector and the path is then

\[ \theta = \tan^{-1} \left( \frac{\partial V}{\partial q} \right) = \tan^{-1} \left( \frac{\partial V}{\partial q} \cdot \frac{\partial q}{\partial q} \right) = \tan^{-1} \left( \frac{(1 + K_p) d}{L} \right); \]

we thus consider the path attraction turn rate along the line perpendicular to the path at distance \( L \) from the goal as a function of \( \theta \),

\[ \omega(\theta) = \lim_{\Delta t \to 0} \left[ \theta - \tan^{-1} \left( \frac{(1 + K_p) d - \Delta t \sin \theta}{L - \Delta t \cos \theta} \right) \right] / \Delta t \]

\[ = \lim_{\Delta t \to 0} \left[ \theta - \tan^{-1} \left( \frac{L \tan \theta - \Delta t (1 + K_p) \sin \theta}{L - \Delta t \cos \theta} \right) \right] / \Delta t \]

\[ = \frac{K_p}{L} \sin \theta \cos^2 \theta. \]

We now proceed to find the angle with maximum turn rate along the line by setting \( \omega'(\theta) = 0 \); hence we have

\[ \theta = \tan^{-1} \frac{1}{\sqrt{3}}. \]

The maximum turn rate at distance \( L \) from the goal is then

\[ \omega(\theta = \tan^{-1}(1/\sqrt{3})) = \frac{K_p \sqrt{2}}{3\sqrt{3}L}. \]

(31)

The closest to the goal that this formula applies is \( L = r_p \), and thereby, the maximum path attraction turn rate is

\[ \frac{K_p \sqrt{2}}{3\sqrt{3}r_p}. \]

We note that for the region within \( r_p \)-distance from the goal, the turn-rate is scaled down to \( \beta(\sqrt{(L^2 + d^2) / r_p}) \frac{K_p \sqrt{2}}{3\sqrt{3}L}. \]

C. The Swirling Parameters \( K_s \) and \( r_s \)

By exploring the adjusted gradient (18) around the moving obstacle, we find a singular point at the boundary of the swirling radius behind the obstacle where the turn-rate becomes unbounded; see Figure 14. This means that by adding swirling effect to the navigation function, the property of being a \( C^2 \)-function has been compromised. This is due to a sudden
change of the gradient to the opposite direction caused by the swirling term. However, we can avoid this region by swirling effect reduction described in § IV-A. Meanwhile, the parameters $K_s$ and $r_{ss}$ also effect the turn-rates inside the swirling zone. As we increase the swirling radius $r_{ss}$, this effect grows at a slower rate, starting from the farther distance to the obstacle, resulting in a lower turn rate for the UAV.

In the contrary, if we increase the relative magnitude of the swirling effect $K_s$, the relation (18) shows that the effect is more emphasized and the aircraft will turn at a higher rate.

**Proposition 5.3:** When a vehicle heads directly toward the moving obstacle, and enters the swirling zone with unit velocity, the turn rate is bounded by $mK_s$, where $m$ denotes the maximum slope of the bump function (12).

**Proof.** If a vehicle heads directly toward the moving obstacle, swirling vectors will be added perpendicular to the original gradient direction as shown in Figure 15. This modification has an effective radius of $r_{ss}$ and maximum relative size of $K_s$ with respect to the norm of the original gradient at the obstacle point and dims to zero at the distance $r_{ss}$, analogous to the bump function (12). Since (12) is normalized to unity, the change in this augmented swirling term is scaled by $K_s$ and $1/r_{ss}$. Thus, the adjusted gradient has the rate bounded by $mK_s r_{ss}$ and the turn rate satisfies

$$\omega \leq \lim_{\Delta t \to 0} \left( \frac{\tan^{-1} \left( \frac{mK_s \Delta t}{\Delta t} \right)}{\Delta t} \right) = \frac{mK_s r_{ss}}{r_{ss}}.\quad \blacksquare$$

The bound given in Proposition 5.3 can not be applied when the vehicle’s heading is not directly toward the moving obstacle. However, the plot in Figure 14(c) suggests that the angles between the original gradients and the added swirling gradients do not deviate much from $90^\circ$ except in the region near the singular point.

**D. Control Gains $K_{\psi_i}$ and $K_{\gamma_i}$**

As we have shown $\psi_i(t) - \psi^d_i(t) \to 0$ as $t \to \infty$, which translates to having $\dot{\psi}_i(t)$ track $\psi^d_i(t)$ well as long as $K_{\psi_i}$ and $K_{\gamma_i}$ are large enough. Moreover $\psi^d_i(t)$ does not undergo a sudden change and

$$\begin{align*}
-\dot{\psi}_{max} &\leq K_{\psi_i} (\psi_i^d(t) - \psi_i(t)) + \dot{\psi}_i(t) \leq \dot{\psi}_{max}, \\
-\dot{\gamma}_{max} &\leq K_{\gamma_i} (\gamma_i^d(t) - \gamma_i(t)) + \dot{\gamma}_i(t) \leq \dot{\gamma}_{max},
\end{align*}$$

(32)

The feedback term $K_{\psi_i} (\psi_i^d - \psi_i)$ and $K_{\gamma_i} (\gamma_i^d - \gamma_i)$ are important only at the beginning when the initial states are set different from the paths or when there are sudden changes to the way-points. The gain $K_{\psi_i}$ and $K_{\gamma_i}$ should be set according to (32). However, setting these gains too low may lead to the slower responses to the small tracking error. Therefore, we choose the control gains such that the turn-rate limits are reached when the error is around $15^\circ$ and impose the saturation to the control commands when the error is larger in the beginning.

Propositions 5.1, 5.2, and 5.3, along with (32) can help landscape the navigation function and adjust swirling effects such that the desired heading command stays within the aircraft turn-rate limits. Proposition 3.3 suggests that the parameter $k$ should be chosen large enough such that the local minima are pushed toward the obstacle. Since the local minima stay inside other vehicle protected zone while the vehicle maneuvers outside other vehicles’ protected zone, we allow the tuning parameter $k$ to be chosen freely since it does not affect the turn rate of the vehicle. For simplification, we choose $k$ such that a the potential value at a state far away from the boundary and the target has a value of 0.5 (according to Figure 4, $k = 0.2$). The parameters $K_p$, $r_p$, $K_s$, and $r_{ss}$, are then chosen based on Propositions 5.2 and 5.3.

**VI. Simulation Examples**

Figure 16 compares the simulation results for the head-on case. The turn-rate limit is set at 0.5 rad/sec. The parameters are chosen based on formulas presented in § VII in order to meet
the turn-rate constraints and guarantee the minimum distance between vehicles: \( k = 0.2, r_p = 20, K_p = 20, K_s = 2, \) and \( r_{ss} = 30. \) Other parameters are set as follow: \( \delta_j = \delta_c = 5, \rho_j = 0, \rho_c = 2000 \) and the control gain \( K_\gamma_0 = K_\gamma_1 = 0.6. \) The bump function (12) is chosen to be the tenth-order polynomial \( f(r_{ij}) = 94.26r_{ij}^{10} - 679.49r_{ij}^8 + 1936.96r_{ij}^6 - 2926.49r_{ij}^4 + 2594.56r_{ij}^2 - 1383.19r_{ij}^0 + 422.61r_{ij}^{-2} - 63r_{ij}^{-4} \) where \( f'(0) = f''(0) = 0 \) for the potential to be twice differentiable. After we reduce the swirling effect according to Figure 8(b), the trajectories no longer have overshoots, the turn rates are down to around 0.2 rad/sec, and the aircraft return to the nominal paths almost 10 seconds faster. Figure 17 depicts the application of this approach on the symmetric case of 3 vehicles from 120 degree apart that fly through the same point. First, the swirling effect is applied constantly at \( r_{ss} = 20 \) to the closest vehicle detected. Then, the parameter \( r_{ss} \) is raised to 30 in order to reduce the turn rate and gradually decrease \( r_{ss} \) to zero once the conflict is resolved. Figure 18 shows a 3D example with 5 UAVs flying through the same coordinates. Figure 19 depicts yet another example of the deconfliction algorithm with two groups of vehicles that have intersecting trajectories.

VII. CONCLUSION

In this paper, we have developed a deconfliction algorithm that guarantees collision avoidance between a pair of constant speed unicycle-type UAVs as well as convergence to the desired destination for each UAV in presence of static obstacles. In addition, we were able to ensure that the UAVs stay close to their nominal paths, and satisfy maximum turn-rate limitations for each UAV throughout the entire mission. The performance of the algorithm has then been validated with respect to key algorithmic parameters and the mission specific restrictions such as turn rate limits and constant UAV velocities. In this avenue, the proposed navigation function has been judiciously landscaped by adjusting the swirling effect such that the vehicles operate within their turn rate limitations. Finally, the collision avoidance analysis has been performed both for the scenario where the swirling effect is dominant, as well as for the case when it is combined with the navigation function.

There are several possible extensions of this work. First, velocity information of the sensed obstacles can be considered in the potential function construction process in order to improve the collision avoiding performance. The second extension involves allowing UAV speed to vary between its stall and maximum speed, as well as making use of the magnitude of the gradients of the navigation function. Including higher fidelity models for UAVs that include drift or dynamic coupling term provides yet another venue for the extension of this work. Path-length calculation can also be included to adjust the speed of each UAV in order the minimize the effect of the deconfliction maneuver on the time of arrival at a particular way point. Moreover, exploring the convergence of the algorithm when \( n \) vehicles come into the range of detection at the same time should be formalized. Finally, examining the robust performance of the proposed deconfliction algorithm is of prime importance for actual multiple UAV missions.

ACKNOWLEDGMENTS

The authors thank Amirreza Rahmani at the University of Miami and Takashi Tsukamaki at the Boeing Company for valuable discussions related to the work presented in this manuscript.

APPENDIX

Proof of Proposition 3.2: The Hessian of \( V_i(q) \) is:

\[
\nabla^2 V_i(q) = \left( \Gamma_i(q)^k + \beta_i(q) \right) \left( k \nabla q_i \beta_i(q) \nabla q_i \Gamma_i(q) \right)^T
\]

\[
-\nabla q_i \Gamma_i(q) \nabla q_i \beta_i(q)^T + k \beta_i(q) \nabla^2 q_i \Gamma_i(q) - \Gamma_i(q) \nabla^2 q_i \beta_i(q)
\]

\[
- \left( \frac{1}{k} + 1 \right) \left[ k \beta_i(q) \nabla q_i \Gamma_i(q) - \Gamma_i(q) \nabla q_i \beta_i(q) \right] \left[ k^{\Gamma_i(q)^k + \beta_i(q)} + \frac{1}{k^{\Gamma_i(q)^k + \beta_i(q)}} \right].
\]

When \( \nabla q_i \Gamma_i(q) = 0 \) and \( \nabla q_i \beta_i(q) = 0 \), the Hessian becomes

\[
\nabla^2 V(q) = \frac{k \beta_i q_i \nabla q_i \Gamma_i(q) - \Gamma_i(q) \nabla^2 q_i \beta_i(q)}{k^{\Gamma_i(q)^k + \beta_i(q)}}.
\]

From (7), let \( \tilde{q} = \frac{\|q_i - q_d_i\|}{r_p} \) and we have

\[
\nabla q_i \Gamma_i(q) = \begin{cases}
2(q_i - q_d_i) + K_p \nabla q_i d_i^2 ; & \text{when } \|q_i - q_d_i\| \geq r_p, \\
2(q_i - q_d_i) + K_p \nabla q_i d_i^2 + f'(\tilde{q}) \frac{d_i^2 q_i - q_d_i}{r_p} ; & \text{when } \|q_i - q_d_i\| < r_p,
\end{cases}
\]

and

\[
\nabla^2 q_i \Gamma_i(q) = \begin{cases}
2I + K_p T \text{diag}(2, 2, 0)^{-1} \frac{\beta_i(q) T \text{diag}(2, 2, 0)^{-1} + \left( f'(\tilde{q}) \frac{d_i^2 q_i - q_d_i}{r_p} \right) f'}{\|q_i - q_d_i\|} ; & \text{when } \|q_i - q_d_i\| \geq r_p, \\
2I + K_p \left( f'(\tilde{q}) \frac{d_i^2 q_i - q_d_i}{r_p} \right) f' \frac{d_i^2 q_i - q_d_i}{r_p} \frac{d_i^2 q_i - q_d_i}{r_p} + \frac{f'(\tilde{q})}{r_p} \left( \frac{d_i^2 q_i - q_d_i}{r_p} \right) \left( \frac{d_i^2 q_i - q_d_i}{r_p} \right)^T ; & \text{when } \|q_i - q_d_i\| < r_p,
\end{cases}
\]

where \( I \) is the identity matrix in \( \mathbb{R}^{3 \times 3} \), \( T \) is a coordinate transformation matrix that transforms a unit vector in the \( z \)-axis to the vector \( q_d_i - q_o_i \), and \( f' \) and \( f'' \) are the first
and second derivatives of the function $f$ defined in (12), respectively. Let $\mathbf{q}$ be a unit vector along the vector difference $\mathbf{q}_i - \mathbf{q}_{d_i}$, and $\xi$ be the angle between $\mathbf{q}_i - \mathbf{q}_{d_i}$, and $\mathbf{q}_{d_i} - \mathbf{q}_{d_i}$; then $\nabla_2 \Gamma_i(\mathbf{q})$ becomes

$$
\nabla_2 \Gamma_i(\mathbf{q}) = \begin{cases} 
2I + \mathbf{K}_r \text{diag}(2, 2, 0) T^{-1} & \text{when } \| \mathbf{q}_i - \mathbf{q}_{d_i} \| \geq r_p, \\
2I + \mathbf{K}_p \frac{\partial^2 (\mathbf{q})}{\partial \mathbf{q} \partial \mathbf{q}} \text{diag}(2, 2, 0) T^{-1} & \text{when } \| \mathbf{q}_i - \mathbf{q}_{d_i} \| < r_p,
\end{cases}
$$

When the $i$-th vehicle reaches its destination that is also far from all other vehicles, we also have $\Gamma_i(\mathbf{q}) = 0$, $\beta_i(\mathbf{q}) = 1$, and $\nabla_2 \Gamma_i(\mathbf{q}) = 0$ simplifies (34) to $\nabla_2 \Gamma_i(\mathbf{q}) = \nabla_2 \Gamma_i(\mathbf{q})$.

Now, by examining the above equation for the case when $\| \mathbf{q}_i - \mathbf{q}_{d_i} \| = 0 < r_p$, we have $\beta(\mathbf{q} = 0) = 0$, $d_i = 0$, and the high order polynomial are chosen such that $f''(0) = f''(0) = 0$ for the potential function to be twice differentiable throughout the configuration space. Hence, $\nabla_2 \Gamma_i(\mathbf{q})$ evaluated at $\mathbf{q}_{d_i}$ equals to $2I$ (positive definite) and hence, the equilibrium point is a minimum. ■

**References**


