Spacecraft Synchronization in the Presence of Attitude Constrained Zones

Unsik Lee and Mehran Mesbahi

Abstract—In this paper, we consider the problem of achieving identical orientation for a group of spacecraft in presence of attitude forbidden zones. The attitude forbidden zones can be identically defined over the group of spacecraft or can be defined independently. In order to find a feasible consensus-like algorithm, we utilize the quadratic convex parameterization for the attitude forbidden zones with unit quaternions. Subsequently, we embed this parameterization into a convex auxiliary system whose output is shared among the spacecraft through a communication network. The auxiliary system's output play the role of an indicator function for the spacecraft dynamics such that it does not violate the attitude forbidden zones while attempting to reach consensus. In order to evaluate the effectiveness of the algorithm, we present two sets of simulations where synchronization of six spacecraft with identical and independent attitude forbidden zones with random initial attitudes are considered.

I. INTRODUCTION

During the past decade, networked dynamic systems have received significant attention from researchers across various fields of science and engineering, in the realm of collective behavior [1], multi-robot control [10], and attitude alignment [11]. The motivation for using networked systems in the space industry is their potential impact on system robustness, flexibility, and scalability. For example, it has been shown that the functionality of the overall space system could dramatically be improved by having distributed systems working together for a number of earth and space science missions. Consensus type coordination algorithms have emerged as a convenient framework to reason about a number of issues surrounding distributed networked dynamic systems. Cooperative control laws among distributed systems using consensus-like protocols generally assume that these systems are modeled as single or double integrators. However, the problem of cooperative control for rigid bodies consists of intricate challenges that are mainly due to its inherent nonlinear dynamics. In [12], fundamental aspects of spacecraft formations have been presented. Recently, a few research works have investigated the relative attitude synchronization for a group of spacecraft. In [13], for example, a behavioral approach has been utilized to maintain attitude alignment between a group of spacecraft interacting over a ring topology. The work [16] has extended this result for more general communication topologies. Moreover, the passivity approach has been adopted in [14] to derive a control law for the spacecraft formation without the angular velocity measurements.

In this paper, we consider the attitude synchronization problem when each spacecraft has its own attitude forbidden zone. This problem appears, for example, in the context of multiple spacecraft interferometry which requires precise target synchronization over multiple spacecraft while avoiding direct exposure to the sunlight or other bright sources due to sensors' sensitivity. Limited number of areas The presented approach is, however, novel as removing constrained zones from the rotational configuration space of the spacecraft generally results in a non-convex domain. The spacecraft synchronization problem in the absence of attitude constrained zone has been comprehensively addressed in the literature since the domain of the rotation group is a closed manifold.

Our approach utilizes a convexified quadratic inequality over unit quaternions in order to describe the respective attitude constrained zones. Subsequently, we introduce an auxiliary system, utilizing a logarithm barrier potential that measures how the current attitude deviates away from the boundary of its constrained zone. This auxiliary variable is then shared with the neighbors of each spacecraft. The fact that the total penalty function for the group can be convexified enables us to apply the constrained consensus algorithm presented in [7]. In the constrained consensus problem, each agent converges to the intersection of the domain sets of its neighbors if the agents values are constrained in convex sets and each agent is only aware of its respective constrained set. We then proceed to show that a consensus-like control derived from this approach can be extended to the case of rigid bodies.

The rest of the paper is organized as follows. §II contains the notation, relevant mathematical background, and a brief overview of graph theory and rigid body dynamics. In §III, we state the problem formulation and introduce the so-called “auxiliary” system. We then proceed to embed this auxiliary system in the continuous constrained consensus algorithm. The convergence analysis is then shown for evolution of the spacecraft states as generated by the proposed constrained consensus algorithm. In order to evaluate the effectiveness of the algorithm, two sets of simulations are presented in §IV. Conclusions and potential future extensions of this work are detailed in §V.

II. PRELIMINARIES

A. Notation and graph theory

In this section, we provide the notation and terminology along with a brief background on graph theory. Throughout the paper, a unit quaternion is considered as a $4 \times 1$ column array and denoted by $q$; it will be represented as $q = \begin{bmatrix} q_0 & q_1 & q_2 & q_3 \end{bmatrix}^T$.
when the link graph \( G \) is directed, node \( i \) can transmit information to node \( j \) when the link \( i \to j \) is present in the graph. An undirected graph \( G \) is represented as a pair \( G = (V, E) \), where \( V \) is a finite nonempty set of nodes as \( V = \{v_1, v_2, \ldots, v_n\} \) and \( E \) is referred to as the set of edges of \( G \) and denoted \( E = \{e_1, e_2, \ldots, e_q\} \). An element of \( E \), e.g., \( e_j \), consists of a pair of distinct nodes, where nodes \( v_i \) and \( v_j \) are connected if \( \{v_i, v_j\} \in E \). Analogously, the neighbors of the \( i \)th node are denoted by \( N_i = \{v_j \in V \mid \{v_i, v_j\} \in E\} \). We say that the connection graph \( G \) is connected if for every pair of distinct nodes in \( V \), there is a path that has them as its end nodes. The graph \( G \) can be represented in terms of matrices. Under the assumption that labels have been associated with the edges in a graph, arbitrarily oriented, the \( n \times q \) incidence matrix \( D(G) \) is defined by

\[
D = [d_{ij}], \quad d_{ij} = \begin{cases} 
1 & \text{if } v_i \text{ is the head of } e_j \\
-1 & \text{if } v_i \text{ is the tail of } e_j \\
0 & \text{otherwise}
\end{cases}
\]

The advantage of using the incidence matrix \( D \) over an adjacency representation is that it holds the orientation (vector) of the connection between two nodes. Another matrix representation of a graph \( G \), used in this paper, is the graph Laplacian, \( L(G) \). The symmetric Laplacian matrix which is defined as \( L(G) = DD^T \) holds the connection information among pairs of nodes. This matrix is a positive semi-definite matrix and has eigenvalues that can be ordered as \( \lambda_1(G) \leq \lambda_2(G) \leq \ldots \leq \lambda_n(G) \), where \( \lambda_1(G) = 0 \). The graph \( G \) is connected if and only if \( \lambda_2(G) > 0 \) [5]. The weighted graph Laplacian associated with the weighted graph \( G = (V, E, w) \) can be formed as

\[
L_w(G) = \frac{1}{2} DWWD^T, \quad (1)
\]

where \( W \) is a diagonal matrix whose elements consist of the numeric weights \( w(e_j) \) corresponding to an edge \( e_j \). It is well-known that the inequality \( \lambda_2(G) > 0 \) still holds valid for weighted connected graphs as long as the weights are positive.

### B. Rigid body dynamics using unit quaternions

The attitude dynamics of a rigid body using unit quaternions with fully actuated body-fixed torque devices can be described by the following kinematic and dynamic equation set [9].

\[
\dot{q}(t) = \frac{1}{2} q(t) \otimes \omega(t), \quad (2)
\]

\[
J \dot{\omega}(t) = -\omega(t) \times J \omega(t) + u(t), \quad (3)
\]

where \( q(t) \) is a unit quaternion representing the attitude of the rigid body, \( \omega(t) = [\omega^T \ 0]^T \in \mathbb{R}^3 \), and \( \omega(t) \in \mathbb{R}^3 \) denotes the angular velocity of the spacecraft in the body frame, \( J = \text{Diag}(J_1, J_2, J_3) \) denotes the body frame aligned inertial matrix of the spacecraft, and \( u(t) \in \mathbb{R}^3 \) represents the control torque about the body axes. A unit quaternion representing a rigid body attitude is defined as \( q = [q_0 \ q_1 \ q_2 \ q_3]^T \in \mathbb{D}_q \). In addition, \( \mathbb{D}_n \) denotes an \( m \times n \) identity matrix. We adopt the notation \( \| q \| \) to denote the standard Euclidean norm; namely, \( \| q \| = \sqrt{q^T q} \).

The communication topology of a network between nodes/agents are represented using a directed graph \( G \), i.e., in such a graph, node \( i \) can transmit information to node \( j \) when the link \( i \to j \) is present in the graph. An undirected graph \( G \) is represented as a pair \( G = (V, E) \), where \( V \) is a finite nonempty set of nodes as \( V = \{v_1, v_2, \ldots, v_n\} \) and \( E \) is referred to as the set of edges of \( G \) and denoted \( E = \{e_1, e_2, \ldots, e_q\} \). An element of \( E \), e.g., \( e_j \), consists of a pair of distinct nodes, where nodes \( v_i \) and \( v_j \) are connected if \( \{v_i, v_j\} \in E \). Analogously, the neighbors of the \( i \)th node are denoted by \( N_i = \{v_j \in V \mid \{v_i, v_j\} \in E\} \). We say that the connection graph \( G \) is connected if for every pair of distinct nodes in \( V \), there is a path that has them as its end nodes. The graph \( G \) can be represented in terms of matrices. Under the assumption that labels have been associated with the edges in a graph, arbitrarily oriented, the \( n \times q \) incidence matrix \( D(G) \) is defined by

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### III. Problem Statement

#### A. Constrained Zone Formulation

We adopt attitude constrained zone formulation from [6].

An attitude Forbidden Zone is a set of spacecraft orientations that cause the sensitive on-board instruments to have direct exposure to bright celestial objects such as the sun. Multiple attitude forbidden zones can be specified with respect to a single instrument bore-sight vector. In this paper, we only consider one such bore-sight for each spacecraft to streamline the presentation. In presence of an attitude forbidden zone, the Attitude Permissible Zone is defined as its complement. Using unit quaternion representation for spacecraft orientation, the attitude permissible zones can be parameterized in the form of quadratic inequalities.

**Proposition 1:** Let the unit quaternion \( q_i \in \mathbb{D}_q \) describe the attitude of the \( i \)th spacecraft whose bore-sight vector \( y_i \) for the instrument, e.g., an interferometer, lies outside of the attitude forbidden zone, i.e., \( \beta_i > \theta_i \) in Fig.1. The attitude permissible zone which is the subset \( \mathbb{P}_i \subseteq \mathbb{D}_q \) satisfying the above condition can be represented as,

\[
\mathbb{P}_i = \{ q_i \subseteq \mathbb{D}_q \mid q_i^T M_i(\theta_i) q_i < 0 \} \quad (4)
\]

with \( M_i(\theta_i) = \begin{bmatrix} A_i & b_i^T \\ b_i & d_i \end{bmatrix} \),

(5)
where $A_i = x_i y_i^T + y_i x_i^T - (x_i^T y_i + \cos \theta_i) I_3$, $b_i = x_i \times y_i$, and $d_i = x_i^T y_i - \cos \theta_i$, with $i = 1, 2, \cdots n$.

Note that the matrix $M_i$ is constant since $\theta_i$ is a given constant. We elaborate on the notation used above. The index $n$ represents the total number of spacecraft in the group, and $i$ designates the $i$th spacecraft in the group. Moreover, $x_i$ denotes the unit vector (specified in the inertial frame) for the object to be avoided, while $y_i$ indicates the unit vector (in the body frame) representing the bore-sight direction of the $i$th spacecraft. The angle $\theta_i$ is the angle about the direction of the constrained object specified by $x_i$. Without loss of generality, the domain of the angle $\theta_i$ for all $i$ is restricted to $[0, \pi]$. Note that as $\theta_i \rightarrow \beta_i$, to $q_i^T M_i q_i \rightarrow 0$; this property permits us to apply log barrier potential on $q_i^T M_i q_i$.

B. Logarithmic barrier auxiliary system

We now discuss observations that will be subsequently used for the convex parameterization of the forbidden zones and their embedding as an auxiliary system in a cost function.

Proposition 2: Let $M_i(\theta_i)$ be the matrix used to represent the attitude permission zones in Eq. (5). Then for all $\theta_i \in [0, \pi)$ and $q \in \mathcal{D}_q$, one has

$$-2 < \lambda_{\min}(M_i(\theta_i)) \leq q_i^T M_i(\theta_i) q_i \leq \lambda_{\max}(M_i(\theta_i)) < 2.$$

(6)

Proof: From Eq. (5), the symmetric matrix $M_i(\theta_i)$ can be written as

$$M_i(\theta_i) = P(\theta_i) - \cos \theta_i I_4$$

(7)

where

$$P(\theta_i) = \begin{bmatrix} x_i y_i^T + y_i x_i^T - (x_i^T y_i + \cos \theta_i) I_3 & x_i \times y_i \\ x_i \times y_i^T & x_i^T y_i \end{bmatrix}.$$ 

(8)

Since vectors $x_i$ and $y_i$ are unit vectors, $P^T P = I_4$. Thus,

$$P v = \lambda_p v, \quad P^T P v = \lambda_p P v, \quad (I_4 - \lambda_p^2 I_4) v = 0,$$

where $\lambda_p$ is an eigenvalue of $P$ and $v$ is the corresponding eigenvector. The eigenvalues of $P$, on the other hand, are given as $\lambda_p = -1, -1, 1, 1$. Note that from Eq. (7), eigenvalues of the matrix $M(\theta)$ are shifted from $\lambda_p$ depending on $\cos \theta$, which assumes values between $-1$ and $1$. Thus, in view of the fact that $|q(t)| = 1$, Eq. (6) follows.

Proposition 3: Let $M_i(\theta_i)$ be the matrix used to represent the Attitude Permissible Zones in Eq. (5) and consider the quaternion subset $\mathcal{D}_{P_i} \subseteq \mathcal{D}_q$ defined in Proposition 1 specified as

$$\mathcal{D}_{P_i} = \{ q \in \mathcal{D}_q \mid q^T M_i(\theta_i) q < 0 \}.$$

Then this set can be represented as the convex set [15],

$$\mathcal{D}_{P_i} = \{ q \in \mathcal{D}_q \mid q^T \tilde{M}_i(\theta_i) q < 2 \},$$

where $\tilde{M}_i(\theta_i) \in S^n$.

Proof: From Proposition 2, it follows that,

$$\lambda_{\min}(M_i(\theta_i)) \leq q_i^T M_i(\theta_i) q_i \leq \lambda_{\max}(M_i(\theta_i)) < 2,$$

(9)

where

$$\lambda_{\min}(M_i(\theta_i)) = q_i^T \tilde{M}_i(\theta_i) q_i < 2.$$ 

(10)

Now, we consider the logarithmic barrier auxiliary system $g_i : \mathcal{D}_{P_i} \rightarrow \mathbb{R}$,

$$g_i(q_i) = -k_1 \log \left( \frac{-q_i^T M_i q_i}{2} \right)$$

(11)

where

$$\mathcal{D}_{P_i} = \{ q_i \in \mathcal{D}_q \mid -q_i^T M_i q_i > 0 \}$$

for $i = 1, 2, \ldots, n$.

C. Single spacecraft reorientation in presence of constraints

Given the unit quaternion parameterization of spacecraft attitude, a number of model-independent feedback controls for the rigid body reorientation may be represented as follows. A qualified strictly convex penalty (cost) function $V(q) \geq 0$ and the linearity of quaternion kinematics, e.g., $\dot{q} = \frac{1}{2} q \times \omega$, enable deriving a globally stable control law on the quaternion domain. In other words, given a cost function $V(q)$ in unit quaternion, let

$$V_i = V + \frac{1}{2} \omega^T J_i \omega.$$ 

(12)

Then, $V_i \geq 0$, $\forall(q, \omega)$ and the time derivative of $V_i$ along Eqs. (2-3) is given by

$$\dot{V}_i = \frac{\partial V}{\partial q^T} \left( \frac{1}{2} \omega \right) + \omega^T J_i \omega$$

(13)

where the operator $\text{Vec}[\cdot]$ denotes the vector part of its argument. By taking

$$u = -\alpha \omega + \frac{1}{2} \text{Vec}\left( \frac{\partial V}{\partial q} \right) \times q,$$

(14)
Therefore, by LaSalle’s invariance principle \[17\], the origin \(q^*\) of the underlying information-exchange network. Note that as \(n, m\) designates the number of edges in the weighted graph Laplacian of the underlying network; note that \(\omega\) is asymptotically stable. Moreover, if \(V\) is radially unbounded in \(q\), the system is globally asymptotically stable.

Now, we propose a cost function \(V_t: (D_P, R^3) \to \mathcal{R}\) for the attitude reorientation in presence of attitude forbidden zones as

\[
V_t = ||q^*_o \otimes q - q_i||^2 g(q) + \frac{1}{2} \alpha \omega^T J \omega
\]

(17)

where \(D_P = \{ q \in D_q | -q^T M q > 0 \}\) and \(g(q)\) is the auxiliary system given in Eq. (9). The term \(||q^*_o \otimes q - q_i||^2 g(q)\) is positive for all \(q\) and has a minimum of zero when \(q = q_o\).

In \[6\], the authors have shown that \(||q^*_o \otimes q - q_i||^2 g(q)\) is indeed convex in \(q\). Therefore, its negative gradient will guide the state \(q\) approach the final desired orientation \(q_o\) as \(V_t\) converges to zero. Moreover, as \(q \to \partial D_q \) or \(\omega \to \infty\), we have \(V_t \to \infty\). According to Proposition 4, we can thereby obtain a model-independent control

\[
u = -\alpha \omega - \text{Vec} \left[ q^*_o \otimes q \sum_{i \neq j} \frac{1}{2} \parallel q_j - q_i \parallel^2 g_i(q_i) \right]
\]

(18)

for a given \(\alpha > 0\).

### D. Attitude Consensus under Attitude Forbidden Zones

In this section, we expand on the previous section’s results for multi-spacecraft scenarios. In \[7\], it has been shown that the cost function associated with the sum of squares of differences between connected agent’s states and convex auxiliary functions Eq. (9) on states, e.g.,

\[
V(q) = \sum_{i} \sum_{j \neq i} \frac{1}{2} \parallel q_j - q_i \parallel^2 g_i(q_i)
\]

(19)

can be strictly convexified in \(q = [q^T_1 \ldots q^T_n]^T\), where \(q_i \in \{ q \in D_q | g_i(q_i) > 0 \}\). The non-negative matrix \(W\) has the form

\[
W = \begin{bmatrix}
g_{n}(x_{n}) & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & g_{m}(x_{m})
\end{bmatrix}
\]

(20)

where \(n, m \in E, l\) designates the number of edges in the graph \(G\) and \(D\) denotes the incidence matrix representing the underlying information-exchange network. Note that as defined in Eq. (9), the fact that \(q_i \to \partial D_P, g_i(q_i) \to \infty\) and \(V(q)\) is convex in \(q\), lead us to conclude that consensus

\[
q_1 = q_2 = \ldots = q_n
\]

(21)

is achieved when \(V(q)\) in Eq. (19) becomes zero; moreover, the consensus value lies at the intersection of individual attitude permissible zones.

Thus the negative gradient of \(V(q)\) in Eq. (19) leads to a distributed agreement protocol on the state dependent network:

\[
-\nabla V = -[DW D^T] q - \frac{1}{2} \begin{bmatrix}
q^T D \partial W_{q_1} D^T q \\
\vdots \\
q^T D \partial W_{q_n} D^T q
\end{bmatrix}
\]

(22)

where \(q = [q^T_1 \ldots q^T_n]^T\) and the notation \(\partial W / \partial q_i\) designates the partial derivative of the diagonal components of the matrix \(W\) with respect to \(q_i\). Component-wise,

\[
-\partial V / \partial q_i = \sum_{j \neq i} (q_j - g_i) g_i - \frac{1}{2} \parallel q_j - q_i \parallel^2 [-\nabla g_i].
\]

(23)

We refer the reader to \[7\] for a detailed derivation for more general cases. We note that Eq. (23) only requires that each agent has knowledge of the states and auxiliary functions’ outputs from its connected neighbors, e.g., \(x_j\) and \(g_j\) with respect to the \(i\)th agent.

The fact that \(V(q)\) in Eq. (19) maintains a positive value if the agents form a connected network, in addition to the convexity of the corresponding Lyapunov function, enable us to apply Proposition 4. Before we proceed with this analysis, we need the following lemma in order to build an appropriate combined cost function.

**Lemma 5:** For all pairs \(q_i\) and \(q_j\), one has

\[
\parallel q_j - q_i \parallel^2 = ||q^*_o \otimes q_i - q_i||^2.
\]

(24)

**Proof:** The proof follows by utilizing the unit quaternion’s properties as

\[
\parallel q^*_o \otimes q_i - q_i \parallel^2 = 2 - 2 q^T (q^*_o \otimes q_i)
\]

\[
= 2 - 2 q^T j q_i
\]

\[
= q^T j q_j + q^T j q_i - 2 q^T j q_i
\]

\[
= \parallel q_j - q_i \parallel^2.
\]

We now propose the total cost function \(V_t: (q, R^3 \times R^n) \to \mathcal{R}\) given as

\[
V_t = \sum_{i} \left[ \sum_{j \neq i} \frac{1}{2} \parallel q_j - q_i \parallel^2 g_i(q_i) + \frac{1}{2} \omega^T J_i \omega_i \right]
\]

(25)

\[
= \frac{1}{2} q^T [D_w W_s D^T] q + \sum_{i} \frac{1}{2} \omega^T J_i \omega_i
\]

(26)

\[
= q^T L_w q + \sum_{i} \frac{1}{2} \omega^T J_i \omega_i
\]

(27)

where \(q_i \in \{ q \in D_P | -q^T M q_i < 0 \}\) for \(i = 1, \ldots, n, q = [q^T_1 \ldots q^T_n]^T, L_w = L_w \times I_4\), and \(L_w\) denotes the weighted graph Laplacian of the underlying network; note that \(|\cdot|_*\) signifies \(|\cdot| \times I_4\), where \(|\cdot|_*\) denotes the Kronecker
Next, consider the following control law for the $i$th spacecraft

$$u_i = -k_w \omega_i + \frac{1}{2} \text{Vec} \left[ \left( \frac{\partial V}{\partial q_i} \right)^* \otimes q_i \right]$$

(28)

where the operator $\text{Vec}[:]$ denotes the vector part of unit quaternion, and the gain $k_w$ is strictly positive. Hence, Eq. (23) leads to

$$\frac{\partial V}{\partial q_i} = \sum_{j \neq i} (g_j + g_i)[q_i - q_j] - \|q_j - q_i\|^2 [M_i q_j - q_i^T M_i q_i].$$

(29)

Now, we rewrite Eq. (28) from Eq. (29) as

$$u_i = -k_w \omega_i - \frac{1}{2} \sum_{j \neq i} \text{Vec} \left[ (g_j + g_i)(q_j^* \otimes q_i) \right]$$

$$+ \frac{\|q_j - q_i\|^2}{q_i^T M_i q_i} (M_i q_j)^* \otimes q_i,$$

(30)

where

$$g_i(q_i) = -k_i \log \left( -\frac{q_i^T M_i q_i}{2} \right)$$

(31)

for $i = 1, 2, \ldots n$.

Note that $\text{Vec}[q_i^* \otimes q_i] = 0$, and the control $u_i$ only requires knowledge of $q_i$, the auxiliary system outputs $q_j$, and unit quaternions from the neighbors of the $i$th spacecraft. This model independent control can stabilize the scenario when $\omega < \omega_{\text{max}}$ for some constant $\omega_{\text{max}}$.

IV. SIMULATION

In this section, we present simulation results for two cases where six spacecraft are required to align their attitudes in presence of attitude forbidden zones. These spacecraft are assumed to carry light sensitive interferometers with a fixed bore-sight along the $z$-body axis. It is further assumed that they share attitude information as well as auxiliary variables through a bidirectional communication network.

In the first case, we consider the scenario where the attitude forbidden zones are identical over all six spacecraft. In the second scenario, we consider the case where each spacecraft has a distinct attitude forbidden zone. Note that in the second case, it is implicitly assumed that the intersection of all permissible zones is non-empty in order to be able to reach a consensus orientation. The communication network is given as a complete graph for case 1 and for case 2, five edges are randomly eliminated from the complete graph while keeping the graph connected. These graphs are shown in Fig. 2. All initial attitudes, shown in Table 1, are randomly selected but they satisfy the attitude permissible zone $D_{\theta_i}$ for all $i$. Although it is generally not necessary that all initial angular velocities are zero, we assume that this condition holds for our simulation examples. Figs. 3 and 4 describe initial attitudes of the multiple spacecraft and the attitude forbidden zones in three dimensional space. Figs. 5 and 7 on the other hand, depict the unit quaternion trajectories for case 1 and case 2, respectively, while Figs. 6 and 8 depict the corresponding angular velocities over time. Note that all spacecraft converge to the same attitude regardless of the configuration of their respective attitude forbidden zones. Figs. 9 and 10 trace the pointing directions of the light sensitive instruments on the cylindrical projection of celestial sphere for case 1 and 2 respectively, where ‘$\cdot$’ denotes initial orientations while ‘$\times$’ denotes desired orientations.

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1 In this paper, we use ‘$\otimes$’ for denoting the Kronecker product in order to avoid confusion with the symbol for quaternion multiplication ‘$\otimes$’.
V. Conclusion

In this paper, we have considered solutions to the problems of single spacecraft reorientation and multiple spacecraft attitude synchronization in presence of attitude forbidden zones. We have introduced an auxiliary system utilizing a logarithmic barrier potential that was subsequently embedded in the constrained consensus algorithm. For the synchronization problem, each spacecraft was assumed to be able to access only auxiliary system outputs and attitude parameters from its neighboring spacecraft. Simulation results were then presented for a six spacecraft formation to evaluate the effectiveness of the proposed distributed control algorithms.

References