Bearing-Compass Formation Control: A Human-Swarm Interaction Perspective

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Formation control is an important problem in robotics. Much work has been done analyzing *distributed* algorithms to acquire formations.

- **Distance-based formation control**
  - Egerstedt and Hu (2001)
  - Olfati-Saber and Murray (2002)
  - Mesbahi and Egerstedt (2010)

- **Bearing-based formation control**
  - Moshtagh, Michael, Jadbabaie, and Daniilidis (2009)
  - Bishop, Shames, and Anderson (2011)
  - Franchi and Giordano (2012)

What if we add a **compass** to bearing-based formation control?
Motivation for Bearing-Compass Formation Control

- The addition of a compass provides a cheap, passive, and global reference source to supplement bearing control.
- Having access to absolute bearing information allows the formation to be oriented against a common reference frame.
- Key to effective human-swarm interaction is incorporating intuitive high-level commands:
  - rotation, translation, scaling
  - scale and centroid invariance
Motivation for Bearing-Compass Formation Control

- The addition of a compass provides a cheap, passive, and global reference source to supplement bearing control.
- Having access to absolute bearing information allows the formation to be oriented against a common reference frame.
- Key to effective human-swarm interaction is incorporating intuitive high-level commands: rotation, translation, scaling, scale and centroid invariance.

The addition of a compass and selective node control facilitates achieving all of these operations.
Problem Statement

Consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $n = |\mathcal{V}|$ agents.

$r_i(t)$: 2D position of agent $i$ at time $t$

$\hat{r}_{ij}(t)$: unit bearing of agent $j \in \mathcal{N}(i)$, w.r.t. agent $i$ and north

$\hat{f}_{ij}(t)$: unit desired direction of agent $j$ w.r.t. agent $i$

$\Theta(\mathcal{G}, t)$: set of all $\hat{f}_{ij}(t)$'s defining the formation, $\{i, j\} \in \mathcal{E}(\mathcal{G})$

Notation

Objective: drive $\hat{r}_{ij}^T \hat{f}_{ij} \rightarrow 1$ or (almost) equivalently $\hat{r}_{ij}^T \hat{f}_{ij}^\perp \rightarrow 0$. 

Schoof, Chapman, and Mesbahi
Bearing-Compass Formation Control
We define a *realizable bearing set* $\Theta$ as one where all the bearing constraints can be met [Bishop et. al. (2011)], i.e.

$$
\chi(\Theta) = \left\{ \begin{bmatrix} \hat{r}_1^T & \hat{r}_2^T & \cdots & \hat{r}_n^T \end{bmatrix}^T : \hat{r}_{ij}^T \hat{f}_{ij}^\perp = 0 \text{ for all } \hat{f}_{ij} \in \Theta \right\} \neq \emptyset
$$
The formation set $\chi(\Theta(t))$ is *parallel rigid* if its elements are unique under scaling and translation.

Non-parallel rigid formation and parallel rigid formation
Parallel Sets - Addressing “almost”

- \( \hat{r}_{ij}^T \hat{f}_{ij}^\perp = 0 \) implies \( \hat{r}_{ij}^T \hat{f}_{ij} = 1 \) (parallel) or \( \hat{r}_{ij}^T \hat{f}_{ij} = -1 \) (anti-parallel)

Parallel formation in \( \chi_s \) and anti-parallel formation

Let the parallel formation set be \( \chi_s \subseteq \chi \) where \( \hat{r}_{ij}^T \hat{f}_{ij} = 1 \) for all \( \{i, j\} \in E \).
Agent Dynamics

Each agent is modeled using single integrator dynamics:

\[
\dot{r}_i(t) = u_i(\Theta(t)) + \tilde{u}_i(t)
\]

\[
u_i(\Theta(t)) = -\sum_{j \in \mathcal{N}(i)} (\hat{r}_{ij}^T \hat{f}_{ij}) \hat{r}_{ij}^\perp
\]

- \(u_i(\Theta(t))\): bearing correction control\(^1\)
- \(\hat{r}_{ij}^T \hat{f}_{ij}\): magnitude of motion, proportional to bearing error
- \(\hat{r}_{ij}^\perp\): direction of motion, agent \(i\) orbits around agent \(j\)
- \(\tilde{u}_i(t)\): external additive control input

Open dynamics example video.

\(^1\)Since \(\|\hat{r}_{ij}^T \hat{f}_{ij}\| \leq 1, \|\dot{r}_i\| \leq |\mathcal{N}(i)|\) when \(\tilde{u}_i = 0\).
Symmetry features of the controller induce the following invariant properties when \( \tilde{u}_i = 0 \), for all \( i \):

**Theorem (Constant Centroid)**

The centroid of the formation remains constant, i.e.,

\[
C(r) := \frac{1}{|N|} \sum_{i \in N} r_i = \begin{bmatrix} c_x \\ c_y \end{bmatrix}
\]

**Theorem (Constant Scale)**

The sum-squared distances along each axis (e.g., \( \sum x^2 \) and \( \sum y^2 \)) to the formation centroid remains constant, i.e.,

\[
S(r) := \| r - C(r) \|_2^2 = s
\]
Let $\xi(r, \Theta) = \{f \in \chi_s(\Theta) : C(r) = C(f) \text{ and } S(r) = S(f)\}$.

**Theorem (Unforced Stability)**

*From initial conditions $r_0$, the equilibrium $\xi(r_0, \Theta)$ is almost globally exponentially stable.*

The rate of convergence is

$$\left(2m\sqrt{S(r_0)}\right)^{-1} \lambda_2 (L(G))^2 \cos^2(\delta)$$

where $r_0 \in D_r(\delta) := \{r \in \mathbb{R}^n : \|r - \xi(r, \Theta)\| \leq 2 \|r\| \sin(\delta)\}$, and $\delta \in [0, \frac{\pi}{2})$.

The rate of convergence improves with:

- the ratio of graph connectivity to edges, $\lambda_2 (L(G))^2 / m$
- smaller formation scale, $S(r_0)$
- alignment with desired formation, $\delta$
Additive Control

Up until now, we have considered unforced dynamics, i.e., $\tilde{u} = 0$.

$$
\dot{r}_i(t) = u_i(\Theta(t)) + \tilde{u}_i(t)
$$

$$
u_i(\Theta(t)) = - \sum_{j \in \mathcal{N}(i)} \left( \hat{r}_{ij}^T \hat{f}_{ij}^\perp \right) \hat{r}_{ij}^\perp
$$

Now we will consider cases when $\tilde{u} \neq 0$, specifically when

- $\tilde{u}_k \neq 0$ for a single agent $k$ (node control)
- $\tilde{u}_i, \tilde{u}_j \neq 0$ for a pair of agents $i$ and $j$ (edge control)
- $\tilde{u}_1 = \tilde{u}_2 = \cdots = \tilde{u}_n \neq 0$ for all agents (broadcast control)
What can we do to the formation if we can add a control to a single agent, i.e., $\tilde{u}_k \neq 0$ for a single agent $k$?

**Proposition (Node Control)**

*Under non-zero additive control $\tilde{u}_k \neq 0$, for a single agent $k$*

\[
\frac{\partial C}{\partial t} = \frac{1}{|N|} \sum_{i \in N} \dot{r}_i = \frac{1}{n} \tilde{u}_k \\
\frac{\partial S}{\partial t} = 2 \frac{n - 1}{n} (r_k - C)^T \tilde{u}_k
\]

Open translation example video.
Open scaling example video.
What can we do to the formation if we can add a control to a pair of neighboring agents?

**Corollary (Pure Scaling)**

*If we command agents $i$ and $j$ to move directly towards or away from one another, the formation will experience a pure scaling.*

This corollary is particularly useful when $\{i, j\} \in E$, since they are aware of the direction $\hat{r}_{ij}$.

Open pure scaling example video.
What can we do if we apply the same control signal to all agents?

**Corollary (Pure Translation)**

*If all agents apply a common constant control, then the formation translates in the direction of the control with no scaling.*

What about broadcast rotation control? Consider

\[
\dot{r}_i(t) = u_i(\Theta(t))
\]

\[
u_i(\Theta(t)) = -\sum_{j \in \mathcal{N}(i)} \left( \dot{r}_{ij}^T \hat{f}_{ij}^\perp(t) \right) \hat{r}_{ij}^\perp
\]

where \( \hat{f}_{ij}(t) \) is the formation vector rotating at a constant rate \( \omega \).

Open broadcast control video.
Consider $\hat{f}_{ij}(t) = R(\theta(t))\hat{f}_{ij}(0)$, where $\dot{\theta}(t) = \omega$ and $R(\theta(t))$ is the 2D rotation matrix.

**Theorem (Rotation Rate)**

The agent equilibrium trajectory is ultimately bounded by

$$b = \frac{4mS(r_0)\omega}{(1-\varepsilon)\lambda_2(L(G))^2\cos^2(\delta)}$$

where $r_0 \in D_r(\delta)$, $\delta \in [0, \frac{\pi}{2})$, and $\varepsilon > 0$ small.

This bound improves with similar trends: larger ratio of graph connectivity to edges, smaller formation scale, and closer alignment to the desired formation.

- The bound improves with slower rotation rate, $\omega$.

If we rotate too fast, the formation cannot be tracked.

Open fast rotation video.
In this research we have:

- established **invariant properties** on centroid and scale
- demonstrated formation convergence relative to **graph features** and initial conditions
- manipulated the formation using additive control to cause **scaling** and **translation**
- applied broadcast **rotation** control to rotate the formation with guaranteed bounds

In the future, we plan to explore:

- a 3D formulation of this problem
- dynamic estimation of key formation parameters, e.g., the formation centroid