Bearing-Compass Formation Control: A Human-Swarm Interaction Perspective

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Introduction

Formation control is an important problem in robotics. Much work has been done analyzing *distributed* algorithms to acquire formations.

- Distance-based formation control
  - Egerstedt and Hu (2001)
  - Olfati-Saber and Murray (2002)
  - Anderson, Yu, Fidan, and Hendrickx (2008)
  - Mesbahi and Egerstedt (2010)
- Bearing-based formation control
  - Moshtagh, Michael, Jadbabaie, and Daniilidis (2009)
  - Bishop, Shames, and Anderson (2011)
  - Franchi and Giordano (2012)

What if we add a **compass** to bearing-based formation control?

Motivation for Bearing-Compass Formation Control

- The addition of a compass provides a cheap, passive, and global reference source to supplement bearing control
- Having access to absolute bearing information allows the formation to be oriented against a common reference frame
- Key to effective human-swarm interaction is incorporating *intuitive* high-level commands
  - rotation, translation, scaling
  - scale and centroid invariance

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The addition of a compass and selective node control facilitates achieving all of these operations.

Problem Statement

Consider a graph $\mathcal{G} = (V, E)$ with $n = |V|$ agents.

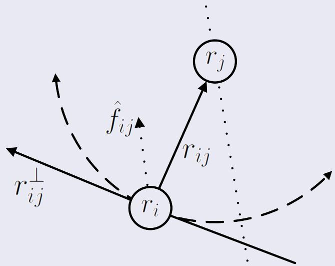
$r_i(t)$: 2D position of agent i at time t

$\hat{r}_{ij}(t)$: unit bearing of agent $j \in \mathcal{N}(i)$, w.r.t. agent i and north

$\hat{f}_{ij}(t)$: unit desired direction of agent j w.r.t. agent i

$\Theta(\mathcal{G}, t)$: set of all $\hat{f}_{ij}(t)$'s defining the formation, $\{i, j\} \in E(\mathcal{G})$

Notation



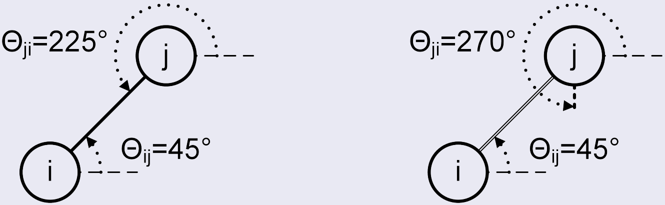
Objective: drive $\hat{r}_{ij}^T \hat{f}_{ij} \to 1$ or (almost) equivalently $\hat{r}_{ij}^T \hat{f}_{ij}^{\perp} \to 0$.

Realizable Bearing Set

We define a *realizable bearing set* Θ as one where all the bearing constraints can be met [Bishop et. al. (2011)], i.e.

$$\chi(\Theta) = \left\{ \begin{bmatrix} r_1^T & r_2^T & \cdots & r_n^T \end{bmatrix}^T : \hat{r}_{ij}^T \hat{f}_{ij}^{\perp} = 0 \text{ for all } \hat{f}_{ij} \in \Theta \right\} \neq \emptyset$$

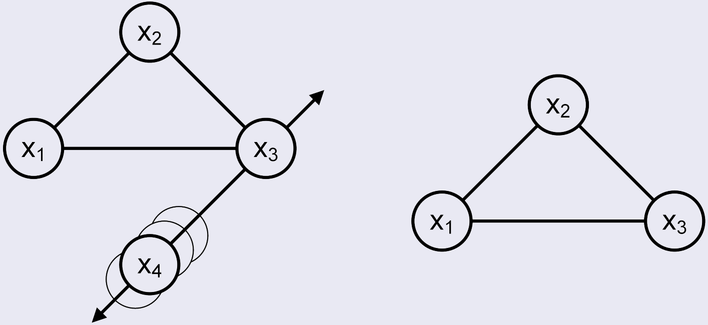
Realizable and non-realizable formation



Parallel Rigidity

The formation set $\chi(\Theta(t))$ is *parallel rigid* if its elements are unique under scaling and translation.

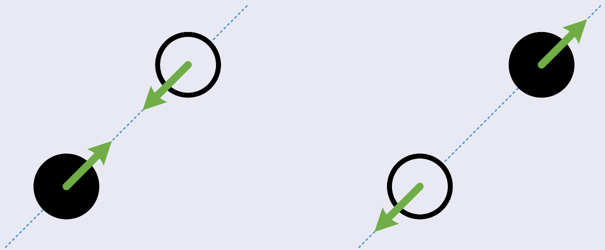
Non-parallel rigid formation and parallel rigid formation



Parallel Sets - Addressing “almost”

- $\hat{r}_{ij}^T \hat{f}_{ij}^{\perp} = 0$ implies $\hat{r}_{ij}^T \hat{f}_{ij} = 1$ (parallel) **or** $\hat{r}_{ij}^T \hat{f}_{ij} = -1$ (anti-parallel)

Parallel formation in χ_s and anti-parallel formation

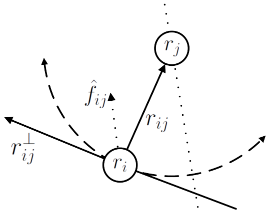


Let the parallel formation set be $\chi_s \subseteq \chi$ where $\hat{r}_{ij}^T \hat{f}_{ij} = 1$ for all $\{i, j\} \in E$.

Agent Dynamics

Each agent is modeled using single integrator dynamics:

$$\begin{aligned}\dot{r}_i(t) &= u_i(\Theta(t)) + \tilde{u}_i(t) \\ u_i(\Theta(t)) &= -\sum_{j\in\mathcal{N}(i)} \left(\hat{r}_{ij}^T \hat{f}_{ij}^{\perp}\right) \hat{r}_{ij}^{\perp}\end{aligned}$$



$u_i(\Theta(t))$: bearing correction control[1]

$\hat{r}_{ij}^T \hat{f}_{ij}^{\perp}$: magnitude of motion, proportional to bearing error

$\hat{r}_{ij}^{\perp}$: direction of motion, agent i orbits around agent j

$\tilde{u}_i(t)$: external additive control input

Open dynamics example video.

[1]Since $\left\|\hat{r}_{ij}^T \hat{f}_{ij}^{\perp}\right\| \leq 1$, $\|\dot{r}_i\| \leq |\mathcal{N}(i)|$ when $\tilde{u}_i = 0$.

Invariant Properties

Symmetry features of the controller induce the following invariant properties when $\tilde{u}_i = 0$, for all i:

Theorem (Constant Centroid)

The centroid of the formation remains constant, i.e.,

$$C(r) := \frac{1}{|N|} \sum_{i \in N} r_i = \begin{bmatrix} c_x \\ c_y \end{bmatrix}$$

Theorem (Constant Scale)

The sum-squared distances along each axis (e.g., $\sum x^2$ and $\sum y^2$) to the formation centroid remains constant, i.e.,

$$S(r) := \| r - C(r) \|_2^2 = s$$

Stability

Let $\xi(r,\Theta) = \{f \in \chi_s(\Theta) \,:\, C(r) = C(f) \text{ and } S(r) = S(f)\}$.

Theorem (Unforced Stability)

From initial conditions r_0, the equilibrium $\xi(r_0,\Theta)$ is almost globally exponentially stable.

The rate of convergence is $\left(2m\sqrt{S(r_0)}\right)^{-1} \lambda_2(L(\mathcal{G}))^2 \cos^2(\delta)$ *where* $r_0 \in D_r(\delta) := \{r \in \mathbb{R}^n \,:\, \|r - \xi(r,\Theta)\| \leq 2\|r\| \sin(\delta)\}$, *and* $\delta \in \left[0, \frac{\pi}{2}\right)$.

The rate of convergence improves with:

- the ratio of graph connectivity to edges, $\lambda_2(L(\mathcal{G}))^2 / m$
- smaller formation scale, $S(r_0)$
- alignment with desired formation, δ

Additive Control

Up until now, we have considered unforced dynamics, i.e., $\tilde{u} = 0$.

$$\begin{aligned} \dot{r}_i(t) &= u_i(\Theta(t)) + \mathbf{\tilde{u}_i(t)} \\ u_i(\Theta(t)) &= -\sum_{j \in \mathcal{N}(i)} \left(\hat{r}_{ij}^T \hat{f}_{ij}^{\perp} \right) \hat{r}_{ij}^{\perp} \end{aligned}$$

Now we will consider cases when $\tilde{u} \neq 0$, specifically when

- $\tilde{u}_k \neq 0$ for a single agent k (node control)
- $\tilde{u}_i, \tilde{u}_j \neq 0$ for a pair of agents i and j (edge control)
- $\tilde{u}_1 = \tilde{u}_2 = \cdots = \tilde{u}_n \neq 0$ for all agents (broadcast control)

Node Control

What can we do to the formation if we can add a control to a single agent, i.e., $\tilde{u}_k \neq 0$ for a single agent k?

Proposition (Node Control)

Under non-zero additive control $\tilde{u}_k \neq 0$, for a single agent k

$$\begin{aligned} \frac{\partial C}{\partial t} &= \frac{1}{|N|} \sum_{i \in N} \dot{r}_i = \frac{1}{n} \tilde{u}_k \\ \frac{\partial S}{\partial t} &= 2 \frac{n-1}{n} (r_k - C)^T \tilde{u}_k \end{aligned}$$

Open translation example video.
Open scaling example video.

Edge Control

What can we do to the formation if we can add a control to a pair of neighboring agents?

Corollary (Pure Scaling)

If we command agents i and j to move directly towards or away from one another, the formation will experience a pure scaling.

This corollary is particularly useful when $\{i, j\} \in E$, since they are aware of the direction $\hat{r}_{ij}$.

Open pure scaling example video.

Broadcast Control

What can we do if we apply the same control signal to all agents?

Corollary (Pure Translation)

If all agents apply a common constant control, then the formation translates in the direction of the control with no scaling.

What about broadcast rotation control? Consider

$$\begin{aligned} \dot{r}_i(t) &= u_i(\Theta(t)) \\ u_i(\boldsymbol{\Theta}(\mathbf{t})) &= -\sum_{j \in \mathcal{N}(i)} \left(\hat{r}_{ij}^T \hat{\mathbf{f}}_{\mathbf{ij}}^{\perp}(\mathbf{t}) \right) \hat{r}_{ij}^{\perp} \end{aligned}$$

where $\hat{f}_{ij}(t)$ is the formation vector rotating at a constant rate ω.
Open broadcast control video.

Constant Rotation Rate

Consider $\hat{f}_{ij}(t) = R(\theta(t))\,\hat{f}_{ij}(0)$, where $\dot{\theta}(t) = \omega$ and $R(\theta(t))$ is the 2D rotation matrix.

Theorem (Rotation Rate)

The agent equilibrium trajectory is ultimately bounded by $b = \frac{4mS(r_0)\omega}{(1-\varepsilon)\lambda_2(L(\mathcal{G}))^2\cos^2(\delta)}$, *where* $r_0 \in D_r(\delta)$, $\delta \in \left[0, \frac{\pi}{2}\right)$, *and* $\varepsilon > 0$ *small.*

This bound improves with similar trends: larger ratio of graph connectivity to edges, smaller formation scale, and closer alignment to the desired formation.

- The bound improves with slower rotation rate, ω.

If we rotate too fast, the formation cannot be tracked.

Open fast rotation video.

Conclusion and Future Work

In this research we have:

- established invariant properties on centroid and scale
- demonstrated formation convergence relative to graph features and initial conditions
- manipulated the formation using additive control to cause scaling and translation
- applied broadcast rotation control to rotate the formation with guaranteed bounds

In the future, we plan to explore:

- a 3D formulation of this problem
- dynamic estimation of key formation parameters, e.g., the formation centroid