Power Management of Cooling Systems with Dynamic Pricing

Saghar Hosseini, Ran Dai, and Mehran Mesbahi

***Abstract*— This paper addresses the optimal power management problems in electric cooling systems based on appropriately constructed thermal dynamic models and cost profiles. In this venue, the dynamics and logical constraints for the cooling load are first formulated as mixed-integer linear programming models. We subsequently apply an online learning algorithm to adjust the weighting factor for customers' satisfaction level considering the fluctuating prices and customers' preferences. The proposed approach is expected to save the user's electricity cost by adequately scheduling the operations of the cooling load without an adverse effect on the entire system. The effectiveness of the proposed temperature control and trade-off between electricity cost and customers' satisfaction level is demonstrated via a simulation scenario.**

***Index Terms*— Power Management; Mixed Integer Programming; Online Learning; Temperature Control; Smart Grid**

I. INTRODUCTION

Future smart grids rely on new levels of transparency and coordination between providers and consumers of electric energy. On one side, the electricity providers will be tracking and estimating the consumers' daily, weekly and seasonal demands to coordinate the energy requests. On the other side, some customers will be enticed to change their consumption habit to save cost and alleviate peak demand by adopting new technologies, e.g., dynamic pricing [1]. The success of cost reduction at the consumer's side requires accessing real-time prices and adapting corresponding strategies, e.g., scheduling operation of loads when the electricity is cheaper. In this paper, we examine an optimization-based approach for temperature control of cooling systems. The goal is to reduce the cost of energy consumption while satisfying the demands of the consumers.

Finding an efficient demand schedule is challenging due to the nonlinear thermodynamics and complex heat exchange processes associated with environment and the heating/cooling systems [2], [3]. Introducing simple thermal load models and applying different control strategies have brought significant progress in this field. For example, Perfumo *et al.* [4] proposed a model based control that thermostatically schedule the loads. Katipamula and Lu [5] evaluated different thermostat setting approaches for energy saving of residential heating, ventilation, and air-conditioning (HVAC) systems. Moreover, Ha *et al.* [6] applied a multi-scale optimization mechanism in load management of electric home heaters. In addition, uncertainties in energy price [7] and power resources [8] affect the resulting optimal control problem. The work presented in [9] addresses the uncertainties in the energy sources by controlling the energy storage and load operation.

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Inspired by these works, we propose a mixed integer linear programming (MILP) model for a thermostatically controlled cooling system which allows for power curtailment strategy to reduce the energy cost. However, our approach can override the thermostat control command when necessary, e.g., temporally pausing the operation of the cooling system during high price intervals without adverse effect on the overall operation. We focus on cooling systems designed specifically for storage of brewery, winery, dairy, and etc., with a preferred temperature range.

By efficiently scheduling the cooling systems' operation, the proposed approach is expected to minimize the energy cost, and at the same time provide a degree of consumer satisfaction which is reflected by the difference between the ideal temperature and real maintained temperature.

The optimization problem is modeled as a trade off between energy cost and the consumer satisfaction level. Therefore, the user's preference and the price he/she is willing to pay has a significant impact on the cost function. However, the electricity price is not known a priori and the user's preference is changing with time, making it difficult to set up a tractable optimization problem. Therefore, an online estimation approach is implemented to model the weighting factor of consumer's satisfaction level. This approach is a special case of our previous work [10] regarding online distributed estimation via dual averaging. Online learning algorithms have been proposed [11], [12] to address the uncertainties in the systems *without probabilistic assumption* where the stochastic optimization methods [13], [14] are inadequate. Such learning algorithms have been widely used to solve the optimization problems with unknown cost function at the time when relevant decision is made [15]. The performance of online algorithms is captured by *regret* which is a standard measure in machine learning literature [16], [17]. *Regret* represents the non-optimality of the algorithm by not following the best fixed decision in hindsight. Consequently, the average regret of a good algorithm should approach zero over time.

The main contribution of the present paper is to formulate an MILP model for the thermostatically controlled cooling systems based on estimated weighting factors obtained from online estimation algorithm and electricity companies. The power curtailment strategy is implemented by solving optimal power management problem via a MILP solver [18].

The organization of the paper is as follows. First, we formulate the power management problem of cooling systems in

§II. Subsequently, the (MILP) model of the thermostatically controlled loads is formulated in §III, followed by the dual averaging algorithm for setting the weighting factor in §IV. Simulation example demonstrating the applicability of the proposed approach is detailed in §V, which is followed by concluding remarks in §VI.

II. PROBLEM FORMULATION

The objective of power management for cooling systems is to schedule the loads' operation such that the consumed electricity cost is minimized and at the same time a degree of consumer satisfaction is maintained. Assuming the consumers have access to the setting the ideal temperature, T_I, for the cooling system, the objective function is defined as

$$\min_{z_1,z_2} \sum_{t=t_0}^{t_f} [c_p(t)(g_1 z_1(t) + g_2 z_2(t)) \\ +c_T|T(t) - T_I|]\Delta t,\ T(t) \in [T_{min}, T_{max}], \quad (2.1)$$

where Δt is the step size of each horizon, g_1 and g_2 are the rapid pull down working power (with fast cooling rate) and the normal working power (with normal cooling rate), respectively. The term c_p refers to the day-ahead predicted electricity price published online from the electricity company at time t. The cooling temperature at horizon t is denoted by $T(t)$, while T_{max} and T_{min} are the allowed highest and lowest temperature, respectively. The term c_T is the coefficient related to the degree of consumer satisfaction; c_T acts as a weighting factor that balances the importance between the electricity cost and the consumer's preference. However, c_T can be adjusted during the control interval when the electricity price or ideal temperature changes in the power system.

The solution of the optimization problem (2.1) is to find the operation control z_1 and z_2, where $z_1,\ z_2 \in \{0, 1\}$ and $z_1(t)+z_2(t) \leq 1$, for rapid pull down or normal operation of every load at each horizon to minimize the objective function from initial time t_0 to final time t_f. During the optimization process, the temperature changes due to the operation and the heat exchange between the system and the environment. The temperature $T(t)$ is constrained by the thermal dynamic equation expressed as

$$T(t + 1) = T_e + Q_1 R z_1(t) + Q_2 R z_2(t) \\ -(T_e + Q_1 R z_1(t) + Q_2 R z_2(t) - T(t)) \exp[-\Delta t/(RC)], \\ z_1(t),\ z_2(t) \in \{0, 1\}, \quad (2.2)$$

where T_e is the temperature of the environment. The electric cooling capacities for rapid pull down and normal operation are denoted by Q_1 and Q_2, respectively. The terms C and R are the cooling thermal capacitance and resistance, respectively.

For the above described power management problem, on one hand, it is desired to maintain the minimum electricity cost based on the predicted tariffs. On the other hand, it is also desired to maintain the degree of consumer satisfaction to keep the cooling temperature as close as possible to the ideal setting. The decision to minimize all the terms in the objective function express a bargaining process that balances between the energy price and the degree of consumer satisfaction. When the predicted tariff is low, we intuitively intend to use the provided electricity to keep the temperature close to the desired setting. Otherwise, the cooling system is scheduled to temporarily shut down to allow the temperature rise above the setting with no power consumption. Then, there is no power consumption and the cost associated to the degree of consumer satisfaction is determined by $c_T|T(t) - T_I|\Delta t$, the third term in (2.1). With the three coupling terms in the objective function (2.1), the goal is to find the best solution to minimize their sum. Therefore, the operation control terms z_1 and z_2 are the key factors in determining the objective value. In addition, considering fluctuating electricity prices and uncertainties of consumers' preference, the weighting factor, c_T, contributes to the objective value as well. Determining the weighting factor is, however, more complicated and requires integration of objective cost from data. In the following, two integrated approaches, MILP and online learning, are introduced to obtain the optimal solution for the power management problem of cooling systems. From the MILP solution, the cooling system finds the controls, z_1 and z_2, under current value of c_T. The online learning approach updates the value of c_T by tracking the time history of the electricity price and ideal temperature settings.

III. TEMPERATURE CONTROL VIA MIXED-INTEGER LINEAR PROGRAMMING

MILP is the optimization problem of minimizing an objective function expressed by a linear combination of integral and real-valued state variables, subject to linear equality and inequality constraints. It can be solved using the branch and bound, branch and cut, or branch and price algorithms. There are numerous applications for MILPs in many areas of operations research, including network flow, path planning, and scheduling.

In order to formulate the MILP model for dynamic programming, the nonlinear term $\exp[-\Delta t/(RC)]$ in (2.2) is approximated as a linear expression $1 - \Delta t/(RC)$. Consequently, the thermal dynamic equation in (2.2) is linearized as

$$T(t + 1) = T(t) + (Q_1 z_1(t) + Q_2 z_2(t))\Delta t/C \\ +(T_e - T(t))\Delta t/(RC). \quad (3.3)$$

During each horizon t, only one operation is allowed among the three options. These options include "rapid pull down" with $z_1(t) = 1$, "normal chilling" with $z_2(t) = 1$, and "off" with $z_1(t) + z_2(t) = 0$. Generally, the rapid pull down has higher cooling rate, but requires more operational power than the normal cooling. The described logical constraints can be expressed as

$$z_1(t) + z_2(t) \leq 1,\ z_1(t),\ z_1(t) \in \{0, 1\},\ \forall\, t \in [t_0, t_f]. \quad (3.4)$$

To find the linear expression of the objective function (2.1), we introduce the new variable $y(t)$ at each time horizon to relax the objective value by assigning $y(t)$ as

$$y(t) \geq |T(t) - T_I|, \quad \forall\, t \in [t_0, t_f].$$

In addition, it is not desirable that the control commands switch frequently between working and "off" states, which is against the healthy operation of the electric unit. Therefore, we assign a upper bound temperature T_b, such that once the the rapid pull down or normal cooling operation is turned off, it will not be turned on again until the temperature increases beyond this upper bound. To realize this constrained operation, a binary variable $z_b(t)$, is introduced to indicate whether the temperature increases beyond the assigned temperature T_b. Logically, z_b satisfies the following constraint

$$z_b(t) - T(t)/T_b \leq 0,\; z_b(t) \in \{0, 1\}, \quad \forall\, t \in [t_0, t_f]. \tag{3.5}$$

The above inequality guarantees that when z_b equals to one, the temperature has to be more than T_b to make the upper bound of $z_b(t) - T(t)/T_b$ less than zero. In other word, $z_b(t)$ cannot be set as one if $T(t) \leq T_b$. However, $T(t) \geq T_b$ does not imply that $z_b(t)$ must be equal to one.

In addition, the following expression,

$$z_1(t) + z_2(t) \leq z_b(t-1) + z_1(t-1) + z_2(t-1) \quad \forall t \in [t_0, t_f],\, t \neq t_0, \tag{3.6}$$

ensures that the two types of cooling operation, rapid pull down and normal cooling, cannot be turned on unless z_b equals one in the previous horizon. The fact that z_b is equivalent to one implies that the boundary temperature is reached, or the cooling operation was performing in the previous horizon. Thus, the cooling operation cannot be turned on if the operation at previous step is not "on", $(z_1(t-1) + z_2(t-1) \neq 1)$, and the temperature does not increase beyond the boundary value $(z_b(t-1) \neq 1)$.

With the two linear constrains expressed in (3.5) and (3.6), we can imagine that once the lower bound temperature is reached, neither of the cooling operations will be on again until the temperature increases beyond the boundary value, that is when $z_b(t) = 1$. The power output for the cooling systems is simply

$$P(t) = g_1 z_1(t) + g_2 z_2(t).$$

From the above description, the MILP formulation for the power management problem described in §II is summarized as

$$\begin{aligned}
&\min_{z_1, z_2, y, z_b} \sum_{t=t_0}^{t_f} [c_p(t)(g_1 z_1(t) + g_2 z_2(t)) + \hat{c}_T y(t)]\Delta t \\
&s.t.\quad T(t+1) = T(t) + (Q_1 z_1(t) + Q_2 z_2(t))\Delta t/C \\
&+(T_e - T(t))\Delta t/(RC),\; T(t) \in [T_{min}, T_{max}], \\
&z_1(t) + z_2(t) \leq 1 \\
&y(t) \geq T(t) - T_I \\
&y(t) \geq T_I - T(t) \\
&z_b(t) - T(t)/T_b \leq 0 \\
&z_1(t) + z_2(t) \leq z_b(t-1) + z_1(t-1) + z_2(t-1) \\
&z_1(t),\, z_2(t),\, z_b(t) \in \{0, 1\}, \forall\, t \in [t_0, t_f].
\end{aligned} \tag{3.7}$$

The above optimization problem provides the operation control z_1 and z_2 and the variables y and z_b for the predicted power price $c_p(t)$, ideal temperature T_I, weighting factor $\hat{c}_T$ and other pre-specified parameters. Most parameters in the above equations are specified from external entities and are time invariant, e.g., g_1 and g_2. However, the electricity price, satisfaction level coefficient, and the ideal temperature are dynamic according to the market and user's preference. Therefore, the power management system needs to adapt the weighting factor $\hat{c}_T$ in order to compensate for these uncertainties. The following section specifically describes an online estimation scheme to predict the weighting factor c_T in (2.1) a day ahead and adjust $\hat{c}_T$ for the MILP (3.7).

IV. Weighting Factor Design via Online Estimation

The consumer satisfaction level coefficient c_T is generally changing based on the user preference and the cost she/he is willing to pay. Therefore, $c_T(t)$ is unknown a priori and an adaptive scheme is required to update this parameter at each time step. If we solve the optimization (2.1) for N days, the cost at the end of day τ based on the actual electricity price $c_{r,\tau}$ is

$$C_R(\tau) = \sum_{t=t_0}^{t_f} [c_{r,\tau}(t)P(t) + c_T \Delta T_\tau(t)]\Delta t, \tag{4.8}$$

while the predicted cost at the end of day τ based on the day-ahead predicted electricity price $c_{p,\tau}$ is

$$C_P(\tau, x) = \sum_{t=t_0}^{t_f} [c_{p,\tau}(t)P(t) + \hat{c}_T \Delta T_\tau(t)]\Delta t. \tag{4.9}$$

Note that $\hat{c}_T(\tau)$ represents the estimated value of c_T on the τth day. Moreover, $\Delta T_\tau(t) = |T(t) - T_I(\tau)|$, and $P(t)$ is the power output for the cooling system. Therefore, specifying consumer satisfaction level coefficient can be formulated as an online estimation problem where the objective is to find the argument x that minimizes the cost function

$$f_\tau(\hat{c}_T) = \frac{1}{2}||C_R(\tau) - C_P(\tau, \hat{c}_T)||_2^2 \;\text{ subject to } \hat{c}_T \in \chi \tag{4.10}$$

Note that $\chi = \{x | 0 \leq x \leq c_{T,\max}\}$ is a closed convex set, where $c_{T,\max}$ is an upper bound on the coefficient representing the consumer's satisfaction level. The function $f_\tau(\hat{c}_T)$ is allowed to change over time in an unpredictable manner due to modeling errors and uncertainties in the electricity price, user preference, and the environment. Based on (4.8) and (4.9), we can express (4.10) as

$$f_\tau(\hat{c}_T) = \frac{1}{2}||b_\tau - \hat{c}_T a_\tau||_2^2 \quad \text{subject to } \hat{c}_T \in \chi, \tag{4.11}$$

where $b_\tau = \sum_{t=t_0}^{t_f} [(c_{r,\tau}(t) - c_{p,\tau}(t))P(t) + c_T(\tau)\Delta T_\tau(t)]\Delta t$ and $a_\tau = \sum_{t=t_0}^{t_f} \Delta T_\tau(t)\Delta t$. The (sub)gradient of the estimation error (4.11) is

$$\partial f_\tau(\hat{c}_T) = -a_\tau(b_\tau - \hat{c}_T a_\tau), \tag{4.12}$$

which is assumed to be known to the pricing algorithm. Further, since the function $f_t(x)$ is convex on a compact domain, it is Lipschitz, i.e. there exists a positive constant L for which

$$|f_\tau(x) - f_\tau(y)| \le L\|x-y\| \quad \text{for all } x, y \in \chi. \qquad (4.13)$$

The Lipschitz constant L for the cost function in (4.11) can be found by observing that [1]

$$\begin{aligned} &|f_\tau(x) - f_\tau(y)| \\ &\le \frac{1}{2}\|a_\tau\|_F(\|a_\tau\|_F\|x-y\|_2 + \|b_\tau\|_2)\|x-y\|_2, \end{aligned}$$

for all $x, y \in \chi$. Assuming that $c_{r,\tau}, c_{p,\tau} \in (0, c_{\max})$, $c_T(\tau) \in (0, c_{T,\max})$, $P \in (0, P_{\max})$, and $\Delta T_\tau \in (0, T_{\max} - T_{\min})$, we have

$$\begin{aligned} L \;&= \tfrac{1}{2}(3c_{T,\max}(T_{\max} - T_{\min}) + 2c_{\max}P_{\max}) \\ &\qquad (T_{\max} - T_{\min})(t_f - t_0)^2. \end{aligned} \qquad (4.14)$$

Note that L is a function of temperature range, maximum power output for the cooling systems, maximum satisfaction level coefficient, and maximum electricity price. Since the energy price, ideal temperature and environment are dynamics, the behavior of (4.10) is unpredictable. Therefore, we need an online scheme in which no assumption or knowledge of the statistical properties of the data are available. In the proposed estimation algorithm, at time step τ, the system estimates $\hat{c}_T \in \chi$ and then an "oracle" announces the cost $f_\tau(\hat{c}_T)$ and its (sub)gradient $\partial f_\tau(\hat{c}_T)$.

A. *Online Estimation via Dual Averaging*

In order to solve this estimation problem in an online setting, we employ Nesterov's dual averaging algorithm [19], [20]. The Online Dual Averaging (ODA) algorithm is presented in Algorithm 1 in which the state variable $\hat{c}_T(\tau)$ and a dual variable $d(\tau)$ are updated sequentially:

$$d(\tau+1) = d(\tau) + \gamma(\tau),$$

where $\gamma(\tau) = \partial f_\tau(\hat{c}_T(\tau))$ is the (sub)gradient of the cost function. Then, $\hat{c}_T(\tau+1) = \prod_\chi^\psi(d(\tau+1), , \alpha(\tau+1))$, where $\prod_\chi(\cdot)$ is a projection onto χ and is defined as

$$\Pi_\chi^\psi(d(\tau), \alpha(\tau)) = \arg\min_{\hat{c}_T(\tau)\in\chi}\{\langle d(\tau), \hat{c}_T\rangle + \frac{1}{\alpha(\tau)}\psi(\hat{c}_T)\}, \qquad (4.15)$$

where $\alpha(\tau)$ is a non-increasing sequence of positive functions. In addition, $\psi(\hat{c}_T) : \chi \to \mathbb{R}$ is a proximal function and is used to avoid wide oscillation in the projection step. Without loss of generality, ψ is assumed to be strongly convex with respect to $\|.\|$,$\psi \ge 0$, and $\psi(0) = 0$. Hence, we can select $\psi(\hat{c}_T) = \frac{1}{k}\|\hat{c}_T\|_2^2$ for some $k > 0$, which is bounded as $\psi(\hat{c}_T) \le \frac{1}{k}c_{T,\max}^2$ in our problem setting. Moreover, Fig 1 and Algorithm 2 present the power management algorithm which is applying the online distributed estimation in the MILP.

[1] Note that the Frobenius norm of A is defined as $||A||_F = \sqrt{\text{trace}(A^TA)}$.

Algorithm 1: Online Dual Averaging (ODA)

for $\tau = 1$ *to* N **do**
  $f_\tau(\hat{c}_T(\tau))$ is revealed
  Compute subgradient $\gamma(\tau) \in \partial f_\tau(\hat{c}_T(\tau))$
  $d(\tau+1) = d(\tau) + \gamma(\tau)$
  $\hat{c}_T(\tau+1) = \prod_\chi^\psi(d(\tau+1), \alpha(\tau))$
end

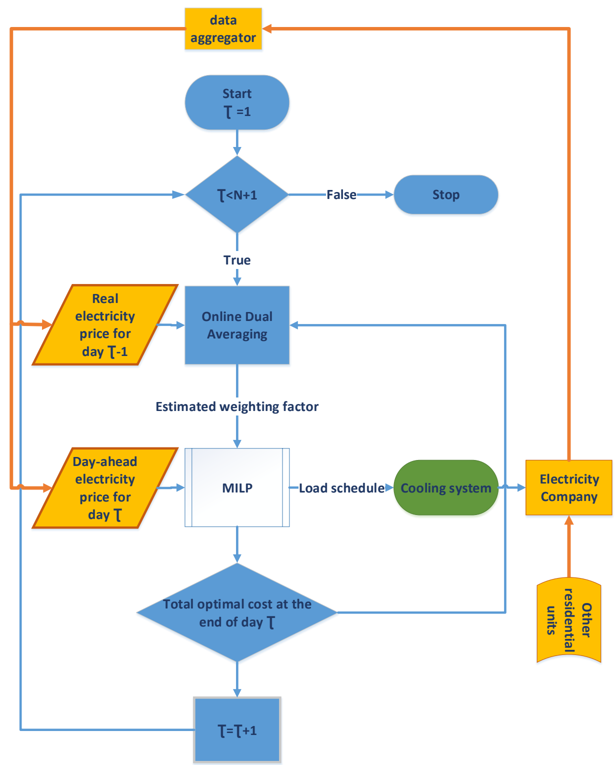

Fig. 1. Power management algorithm.

Algorithm 2: Power management algorithm

for $\tau = 1$ *to* N **do**
  $f_\tau(\hat{c}_T(\tau))$ is revealed
  Compute subgradient $\gamma(\tau) \in \partial f_\tau(\hat{c}_T(\tau))$
  $d(\tau+1) = d(\tau) + \gamma(\tau)$
  $\hat{c}_T(\tau+1) = \prod_\chi^\psi(d(\tau+1), \alpha(\tau))$
  for $t = t_0$ *to* $t = t_f$ **do**
    $C_P(\tau) = \min_{z_1,z_2,y,z_b} \sum_{t=t_0}^{t_f}[c_p(t)P(t) + \hat{c}_T(\tau+1)y(t)]\Delta t$
    subject to constraints
  end
end

B. Convergence Analysis

The convergence analysis for the online algorithms is based on a measure called *regret* and is expressed as

$$R_N(c_T^*, \hat{c}_T) = \sum_{\tau=1}^{N} (f_\tau(\hat{c}_T(\tau)) - f_\tau(c_T^*),) \tag{4.16}$$

over $t = 1, 2, ..., N$, iterations. The goal of online algorithms is to ensure that the cumulative penalty (4.16) that algorithm incurs due to its decisions on the cost sequence $\{f_\tau\}_{\tau=1}^N$ is small. In other words, if $\lim_{N\to\infty} R_N/N = 0$, the algorithm performs as well as the best fixed strategy c_T^* independent of the uncertainties. Further discussion on online algorithms and their regret bound can be found in [15], [17].

The following Lemma IV.1 can be found in [21], and is required for our analysis.

Lemma IV.1. *For any positive and non-increasing sequence $\alpha(\tau)$ and $c_T^* \in \chi$*
$\sum_{\tau=1}^N \langle \gamma(\tau), \hat{c}_T(\tau) - c_T^* \rangle \leq \frac{1}{2}\sum_{\tau=1}^N \alpha(\tau-1)\|\gamma(\tau)\|_*^2 + \frac{1}{\alpha(N)}\psi(c_T^*)$,
where $||.||_*$ is the dual norm. [2]

Theorem IV.2. *The sequence of $\hat{c}_T(\tau)$ and $d(\tau)$ are generated by lines* 4 *and* 5 *in* Algorithm 1*, with $\psi(c_T^*) \leq \frac{1}{k}c_{T,\max}^2$ and $\alpha(\tau) = q/\sqrt{\tau}$, where $k, q > 0$. Thus, we have*

$$R_N(c_T^*, \hat{c}_T) \quad \leq (qL^2 + \tfrac{1}{qk}c_{T,\max}^2)\sqrt{N} - \tfrac{L^2}{2}q,$$

where L is given in (4.14)*.*

Proof: The proof is similar to the regret analysis in [19], [20] and is presented here for completeness. An arbitrary fixed decision $c_T^* \in \chi$ and sequence $\hat{c}_T(\tau)$ generated by line 5 in Algorithm 1 are given. Since f_t is convex,

$$f_\tau(\hat{c}_T) - f_\tau(c_T^*) \leq \langle \gamma(\tau), \hat{c}_T(\tau) - c_T^* \rangle \tag{4.17}$$

Thus, from the definition of regret in (4.16),

$$R_N(c_T^*, \hat{c}_T) \leq \sum_{\tau=1}^{N} (\langle \gamma(\tau), \hat{c}_T(\tau) - c_T^* \rangle). \tag{4.18}$$

Therefore, using the bound in Lemma IV.1, we have

$$R_N(c_T^*, \hat{c}_T) \leq \frac{1}{2}\sum_{\tau=1}^{N} \alpha(\tau-1)\|\gamma(\tau)\|_*^2 + \frac{1}{\alpha(N)}\psi(c_T^*).$$

Note that convexity of f_τ implies that $\langle \gamma, x-y \rangle \leq f_\tau(x) - f_\tau(y)$ for all $x, y \in \chi$. Therefore, based on L-Lipschitz continuity of f_τ, we have $||\gamma||_* \leq L$. Thus, using (4.19) the regret is further bounded as

$$R_N(c_T^*, \hat{c}_T) \quad \leq \tfrac{L^2}{2}\sum_{\tau=1}^N \alpha(\tau-1) + \tfrac{1}{\alpha(N)}\psi(c_T^*),$$

and the theorem follows by applying the integral test on the first term in (4.19). [3]

The "good" performance of the ODA algorithm is demonstrated by sub-linear regret. In addition, the result shows the importance of the underlying system properties through parameter L. Further, we can improve the regret bound by selecting appropriate values for q and k in (3.7).

Next we exhibit a similar dependence on the parameters of the system for the regret analysis of the (temporal) running average estimates.

Corollary IV.3. *The sequence of $\bar{c}_T(\tau)$ is defined as $\bar{c}_T(\tau) = \frac{1}{\tau}\sum_{s=1}^{\tau}\hat{c}_T(s)$ where the sequence of $\hat{c}_T(\tau)$ is generated by line* 5 *in* Algorithm 1*. Let $\psi(c_T^*) \leq \frac{1}{k}c_{T,\max}^2$ and $\alpha(\tau) = q/\sqrt{\tau}$, where $k, q > 0$. Thus, we have*

$$R_N(c_T^*, \bar{c}_T) \leq 2(qL^2 + \frac{1}{qk}c_{T,\max}^2)\sqrt{N}.$$

Proof: Since the cost function $f_\tau(\hat{c}_T(\tau))$ is convex, $f_\tau(\bar{c}_T(\tau)) \leq \frac{1}{\tau}\sum_{s=1}^{\tau} f_\tau(\hat{c}_T(s))$. Therefore, we have

$$f_\tau(\bar{c}_T(\tau)) - f_\tau(c_T^*) \leq \frac{1}{\tau}\sum_{s=1}^{\tau}(f_\tau(\hat{c}_T(s)) - f_\tau(c_T^*)), \tag{4.19}$$

and given the definitions of regret in (4.16) we have

$$R_N(c_T^*, \bar{c}_T) \leq \sum_{\tau=1}^{N}(\frac{1}{\tau}R_\tau(c_T^*, \hat{c}_T)), \tag{4.20}$$

and the corollary follows from (4.20).

V. Simulation Example

In order to illustrate the feasibility of the proposed approach and the MILP model, a simulation example for a small scale cooling system is examined in this section. We aim at validating the applicability and efficiency of the proposed approach for scheduling the operation of the cooling load. At the beginning of each day, we obtain the day-ahead predicted prices online from [7] and calculate the optimal schedules using the MILP algorithm with current ideal temperature setting and the estimated weighting factor. At the end of that day, the real-time price is published online and we adjust the weighting factor for the next day. This process is repeated as long as the operation schedule is required. Additional loads can also be easily included in the current framework. The corresponding parameters of the cooling load used in the simulation are listed in Table I.

TABLE I
PARAMETERS USED IN THE COOLING LOADS

g_1 (kw)	g_2 (kw)	T_e (F)	T_{min} (F)	T_{max} (F)
75	50	72	40	72
$\frac{Q_1}{C}$ (F/s)	$\frac{Q_2}{C}$ (F/s)	$\frac{1}{RC}$ (F/s)	Δt(minutes)	T_b (F)
-3	-1.5	0.02 [20]	2	$T_I + 5$

In addition, the ODA algorithm was applied on the described system to estimate a scalar $c_T \in (0, 0.1)$ with $q = 0.004$, $k = 1$, and $(T_{\max}, T_{\min}) = (40, 72)$. Thus, $\chi = (0, 0.1)$, $c_{T,\max} = 0.1$, and $L = 4953.6$. During one day and night, the MILP algorithm generates optimal schedules for every hour with time step of two minutes. This demand

[2]Note that the dual norm of a vector x is defined as $||x||_* = \sup_{||y||=1} \langle x, y \rangle$.

[3]Note that $\sum_{\tau=1}^N \frac{q}{\sqrt{\tau}} \leq 2q\sqrt{N} - q$.

scheduling is based on the predicted price, the current ideal temperature setting, and the designed weighting factor. The performance of the proposed approach is compared with thermostat strategy based on temperature tracking error [4] and electricity cost. We assume that the thermostat strategy is to switch on when the temperature is above 5^oF over T_I and switched off when it drops 5^oF below T_I.

The temperature tracking error over a month is illustrated in Fig 2 for both online MILP and the thermostat strategy. This figure shows that the temperature profile with the online MILP approach traces the ideal setting better than the one with the thermostat schedule. In addition, Fig 2 depicts a large error on the first day which was improved showing the fast learning rate of the online algorithm.

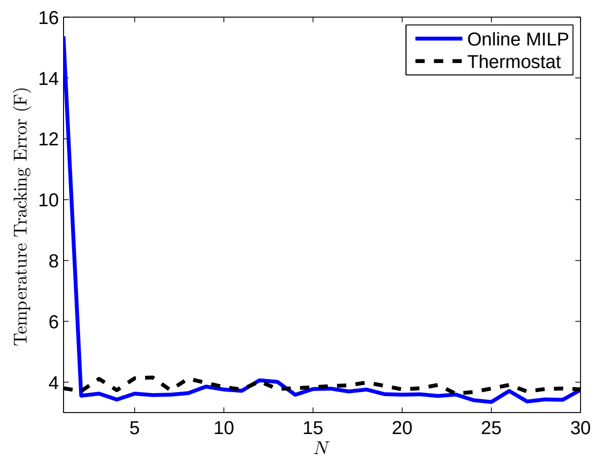

Fig. 2. Temperature tracking error of chilling load with online MILP and thermostat schedule for a month.

Moreover, a comparison between the online MILP and thermostat schedule costs is presented in Figure 3. The

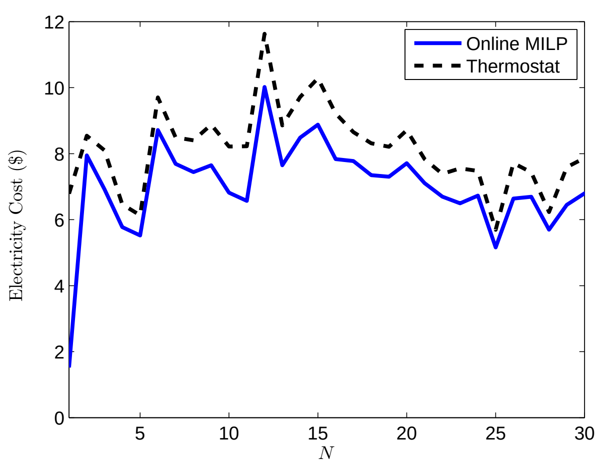

Fig. 3. Electricity cost of each day.

simulation results confirm that the cost for the online MILP scheduling scheme is lower than the the thermostat schedule for the whole month. However, for a cooling system with ten or hundred of similar loads, the saving of electricity cost by adopting the proposed approach is significant. Since the ideal temperature is changing randomly every hour and the real-time price is not known a priori, the user can chose the weighting factor c_T randomly from a uniform distribution. However, $\hat{c}_T$ does not track the user's specified c_T as shown in Fig 4. Despite ignoring user's choice, Figs 3, 2, and 4 imply that designing the weighting factor using online DA algorithm significantly improves the overall performance of the system.

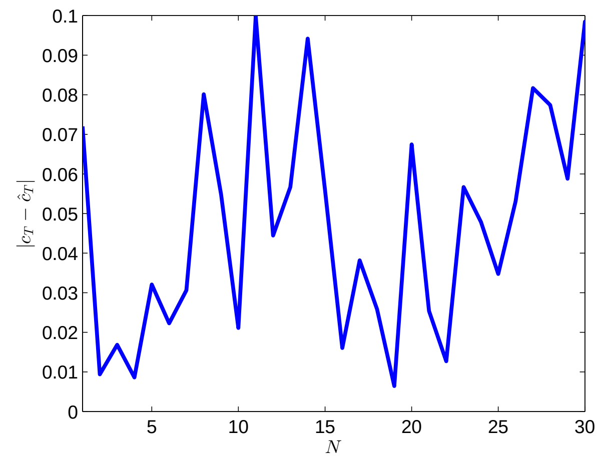

Fig. 4. The time history between dynamic satisfaction coefficient and the constant satisfaction coefficient.

VI. Conclusion

This paper presents an optimal power management strategy in cooling systems in the presence of unknown uncertainties. Blending ideas from online learning and MILP is essential to control the cooling system operation. In this venue, the paper describes an optimization-based modeling technique for thermostatically controlled cooling load via MILP. Moreover, the weighting factor is adjusted as the real cost is revealed. Simulation results show that the online MILP significantly improves the system performance by accommodating for unknown parameters through designing weight factor. Therefore, integrating optimal power curtailment strategies and the proposed algorithm in future smart grid systems can potentially lead to improved operation of the grid in presence of uncertainties.

Future research will consider more complex systems including the operation of multiple loads in the cooling systems.

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