An Online Dual-Averaging Mixed-Integer Programming Approach for Power Management

Saghar Hosseini, Ran Dai and Mehran Mesbahi

Abstract— **Efficient scheduling of power generation, distribution, and usage are crucial for current and future smart grids. Improving the energy management in grid applications will benefit the wholesale electricity market as well as the consumers. This paper addresses the optimal power management problems in electric cooling systems based on appropriately constructed thermal dynamic models and cost profiles. In this venue, the dynamics and logical constraints for the cooling load are first formulated as mixed-integer linear programming models. We subsequently apply an online learning algorithm to adjust the weighting factor for customers' satisfaction level considering the fluctuating prices and customers' preference. The proposed approach is expected to save the electricity cost by adequately scheduling the operations of the cooling load without an adverse effect on the entire system. The effectiveness of temperature control and balance settings between electricity cost and customers' satisfaction level is demonstrated via a simulation using the proposed approach.**

Index Terms— **Power Management; Mixed Integer Programming; Online Learning; Temperature Control, Smart Grid**

I. Introduction

Future smart grids rely on new levels of transparency and coordination between providers and consumers of electric energy. At one side, the electricity providers are tracking and estimating the consumers' daily, weekly and seasonally demands to coordinate the energy requests. At the other side, some customers are willing to change their consuming habit to save cost and alleviate peak demand by adopting new technologies, e.g., dynamic pricing [1], [2]. The success of cost reduction at the consumer's side requires accessing real-time price and taking corresponding strategies, e.g., scheduling operation of loads during time intervals with low price. In this paper, we examine an optimization-based formalism for temperature control of cooling systems to reduce the cost of energy consumption while satisfying the demands from consumers.

The efficient schedule for operation of heating or cooling loads is challenging due to the fact that thermodynamics, generally in the nonlinear form, of the controlled object is often included when considering heat exchange of object with the environment and the heating/cooling systems [3], [4]. By simplifying the thermal load model and applying different control strategies, research in the area has achieved impressive progress. For example, Perfumo *et al.* [5] proposed a model based control to thermostatically control the loads. Katipamula and Lu [6] evaluated different thermostat setting approaches for energy saving of residential heating, ventilating, and air-conditioning (HVAC) systems. Moreover, Ha *et al.* [7] applied a multi-scale optimization mechanism in load management of electric home heaters. When real-time pricing [8], [9] or uncertainties of energy sources [10] are considered in evaluating the power system efficiency, the consumers need to consider the fluctuating prices issued by the electricity company or the available amount when scheduling power requests. For example, the work presented in [11]–[13] has proposed strategies to control the energy storage and load operation including uncertainties in the energy sources .

Inspired by these works, we propose a mixed integer linear programming (MILP) model for a thermostatically controlled cooling system which allows for the power curtailment strategy to reduce the energy cost. We focus on cooling systems designed specifically for storage of brewery, winery, dairy, and etc., with a preferred temperature range. Different from the approaches introduced previously in the literature, our approach can override the thermostat control command when necessary, e.g., temporally pausing the operation of the cooling system during high price intervals without adverse effect. By efficiently scheduling the cooling system operation, the proposed approach is expected to minimize the energy cost, and at the same time provide a degree of consumer satisfaction which is reflected by the difference between the ideal temperature and real temperature in this paper.

Since the optimization problem is modeled as a trade off between energy cost and the consumer satisfaction level, the user's preference and the price he/she is willing to pay has a significant impact on the cost function. However, the electricity price is not known a priori and the user's preference is changing with time, which make it difficult to set up a tractable optimization problem. When the probability distribution of uncertain variables are given, stochastic framework [14]–[16] can be used to improve the robustness of the algorithm. Despite its many successes, the stochastic optimization method fails at addressing the uncertainties in the systems without probabilistic assumption. Online learning algorithms have been proposed [17], [18] for the optimization problems with unknown cost function at the time when relevant decision is made. These algorithms have had a significant impact on modern machine learning

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[19], [20] and have been widely used in real world problems. *Regret* is a standard measure of the performance of online algorithms and represents the difference between the incurred cost and the cost of the best fixed decision in hindsight [21]. Consequently, the average regret of a good algorithm approaches zero.

The online estimation has not been studied at large by the systems and control community. Hosseini *et al.* [22] have proposed an online distributed estimation in a sensor networks. This paper implements an estimation approach for a single refrigeration system to model the weighting factor of consumer's satisfaction level based on dual averaging. For most electricity companies with real-time pricing (RTP) service, the predicted prices is published online one day ahead. The predicted price is indeed an estimation based on the expected consumption of the next day [8]. Then, at the end of that day the true value of the price is revealed as actual charges. On the other hand, the consumer may have time varying preference on the ideal temperature. Therefore, the estimation algorithm attempts to find the weighting factor between the electricity cost and consumer's satisfaction level. Thus, based on the objective function with updated weighting factor, the MILP algorithm will schedule the load of electric cooling systems to minimize the consumed energy cost and at the same time, improve the degree of consumer satisfaction.

Therefore, the main contribution of the present paper is to formulate an MILP model for the thermostatically controlled cooling systems associated with its appropriate cost profile. The power curtailment strategy is implemented by solving optimal power management problem via the MILP solver [23]. In addition, we propose a novel approach to estimate the weighting factor of customer's satisfaction level based on the dual averaging algorithm. Further analysis on the convergence of the proposed approach is provided.

The organization of the paper is as follows. First, we formulate the power management problem of cooling systems in §II. Subsequently, the (MILP) model of the thermostatically controlled loads is formulated in §III, followed by the dual averaging algorithm for setting the weighting factor in §IV. Simulation example demonstrating the applicability of the proposed approach is detailed in §V, which is followed by concluding remarks in §VI.

II. PROBLEM FORMULATION

The objective of power management for the cooling systems is to schedule the loads operation to control the temperature such that the consumed electricity cost is minimized and at the same time the degree of consumer satisfaction is maintained. Assume the consumers have access to the setting of the ideal temperature, T_I, of the cooling system. The objective function is determined by

$$\min_{z_1,z_2} \sum_{t=t_0}^{t_f} [c_p(t)(g_1 z_1(t) + g_2 z_2(t)) \\ +c_T|T(t) - T_I|]\Delta t, \; T(t) \in [T_{min}, T_{max}], \quad (2.1)$$

where Δt is the step size of each horizon, g_1 and g_2 are the rapid pull down working power (with fast cooling rate) and the normal working power (with normal cooling rate), respectively. The term c_p refers to the day-ahead predicted electricity price published online from the electricity company at time t. The cooling temperature at horizon t is denoted by $T(t)$, while T_{max} and T_{min} are the allowed highest and lowest temperature, respectively. The term c_T is the coefficient related to the degree of consumer satisfaction. c_T is acting as a weighting factor which balance the importance between the electricity cost and the consumer's preference, as demonstrated in Fig. (1).

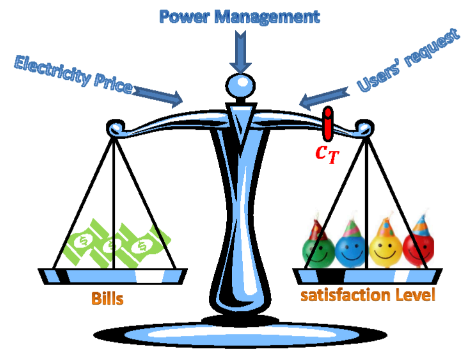

Fig. 1. Bargain between electricity cost and satisfaction level with weighting factor c_T

The objective is to find the operation control z_1 and z_2, where $z_1, z_2 \in \{0,1\}$ and $z_1(t) + z_2(t) \leq 1$, for rapid pull down or normal operation of every load at each horizon to minimize the above function (2.1) from initial time t_0 to final time t_f. During the optimization process, the temperature changes due to the operation and the heat exchange between the cooling system and the environment. The temperature $T(t)$ is constrained by the thermal dynamic equation expressed as

$$T(t+1) = T_e + Q_1 R z_1(t) + Q_2 R z_2(t) \\ -(T_e + Q_1 R z_1(t) + Q_2 R z_2(t) - T(t)) \exp[-\Delta t/(RC)], \\ z_1(t),\; z_2(t) \in \{0,1\}, \quad (2.2)$$

where T_e is the temperature of the environment. The electric cooling capacities for rapid pull down and normal operation are denoted by Q_1 and Q_2, respectively. The terms C and R are the cooling thermal capacitance and resistance, respectively.

For the above described power management problem, on one hand, it is desired to maintain the minimum electricity cost based on the predicted tariffs. On the other hand, it is also desired to maintain the degree of consumer satisfaction to keep the cooling temperature as close as possible to the ideal setting. The decision to minimize all of the terms in the objective function is a bargaining process that balances between the power price and the degree of consumer satisfaction. When the predicted tariff is low, we intuitively intend to use the provided electricity to keep the temperature close to the ideal setting. Otherwise, the cooling system is scheduled to temporarily shut down to allow the temperature rises above the setting with no power consumption. Then, there is no

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power consumption and the cost associated to the degree of consumer satisfaction is determined by $c_T|T(t) - T_I|\Delta t$, the third term in (2.1). With the three coupling terms in the objective function (2.1), the goal is to find the best solution to minimize their summation. Therefore, the operation control terms z_1 and z_2 are the key factors in determining the objective value. In addition, considering fluctuating electricity prices and uncertainties of the consumers' preference, the ideal temperature contributes to the objective value as well. Determining the ideal temperature is, however, more complicated and requires integration of objective cost from pervious experiences. In the following, two integrated approaches, MILP and online learning, are introduced to obtain the optimal solution for the power management problem of the cooling systems.

III. Temperature Control via Mixed-Integer Linear Programming

MILP is the optimization problem of minimizing an objective function expressed by a linear combination of integral and real-valued state variables, subject to linear equality and inequality constraints. It can be solved using the branch and bound, branch and cut, or branch and price algorithms. There are numerous applications for MILPs in many areas of operations research, including network flow, path planning, and scheduling.

In order to formulate the MILP model, the nonlinear term $\exp[-\Delta t/(RC)]$ in (2.2) is approximated as a linear expression $1 - \Delta t/(RC) + h.o.t.$. Consequently, the thermal dynamic equation in (2.2) is linearized as

$$\begin{aligned} T(t+1) = \quad & T(t) + (Q_1 z_1(t) + Q_2 z_2(t))\Delta t/C \\ & + (T_e - T(t))\Delta t/(RC). \end{aligned} \tag{3.3}$$

During each horizon t, only one operation is allowed among the three options. The options include "rapid pull down" with $z_1(t) = 1$, "normal chilling" with $z_2(t) = 1$, and "off" with $z_1(t) + z_2(t) = 0$. Generally, the rapid pull down has higher cooling rate, but requires more operation power than the normal cooling. The described logical constraints can be expressed as

$$z_1(t) + z_2(t) \le 1,\ z_1(t),\ z_1(t) \in \{0,1\},\ \forall\, t \in [t_0, t_f]. \tag{3.4}$$

To find the linear expression of the objective function (2.1), we introduce the new variable $y(t)$ at each time horizon to relax the absolute objective value by assigning $y(t)$ as

$$y(t) \ge |T(t) - T_I|, \quad \forall\, t \in [t_0, t_f].$$

In addition, it is not expected that the control commands switch frequently between working and "off" states, which is against the healthy operation of the electric unit. Therefore, we assign a top boundary temperature, T_b, such that once the the rapid pull down or normal cooling operation is turned off, it will not be turned on again until the temperature increases beyond this top boundary line. To realize this purpose, a binary variable, $z_b(t)$, is introduced to indicate whether the temperature increases beyond the assigned top line temperature T_b. Logically, z_b satisfies the following constraint

$$\begin{gathered} -1 \le z_b(t) - T(t)/T_b \le 0,\ z_b(t) \in \{0,1\}, \\ \forall\, t \in [t_0, t_f]. \end{gathered} \tag{3.5}$$

The above inequality guarantees that when z_b equals to one, the temperature has to be more than T_b to make the upper bound of $z_b(t) - T(t)/T_b$ less than zero. In other word, $z_b(t)$ cannot be set as one if $T(t) \le T_b$. However, $T(t) \ge T_b$ does not imply that $z_b(t)$ must be equal to one.

In addition, the following expression,

$$\begin{gathered} z_1(t) + z_2(t) \le z_b(t-1) + z_1(t-1) + z_2(t-1) \\ \forall t \in [t_0, t_f],\ t \ne t_0, \end{gathered} \tag{3.6}$$

claims that the two types of cooling operation, rapid pull down and normal cooling, cannot be turned on unless z_b equals one in the previous horizon. The fact that z_b is equivalent to one implies that the boundary temperature is reached, or the cooling operation was performing in the previous horizon. Thus, the cooling operation cannot be turned on if the operation at previous step is not "on", $(z_1(t-1) + z_2(t-1) \ne 1)$, and the temperature does not increase beyond the boundary line $(z_b(t-1) \ne 1)$.

With the two linear constrains expressed in (3.5) and (3.6), we can imagine that once the bottom temperature is reached, neither of the cooling operations will be on again until the temperature increases beyond the boundary line, that is when $z_b(t) = 1$. The power output for the cooling systems is simply

$$P(t) = g_1 z_1(t) + g_2 z_2(t).$$

From the above description, the MILP formulation for the power management problem described in §II is summarized as

$$\begin{gathered} \min_{z_1, z_2, y, z_b} \sum_{t=t_0}^{t_f} \left[c_p(t)(g_1 z_1(t) + g_2 z_2(t)) + c_T y(t)\right]\Delta t \\ s.t. \quad T(t+1) = T(t) + (Q_1 z_1(t) + Q_2 z_2(t))\Delta t/C \\ +(T_e - T(t))\Delta t/(RC),\ T(t) \in [T_{min}, T_{max}], \\ z_1(t) + z_2(t) \le 1 \\ y(t) \ge T(t) - T_I \\ y(t) \ge T_I - T(t) \\ -1 \le z_b(t) - T(t)/T_b \le 0 \\ z_1(t) + z_2(t) \le z_b(t-1) + z_1(t-1) + z_2(t-1) \\ z_1(t),\ z_2(t),\ z_b(t) \in \{0,1\},\ \forall\, t \in [t_0, t_f]. \end{gathered}$$

The above optimization problem provides the operation control z_1 and z_2 and the variables y and z_b for the predicted power price $c_p(t)$, ideal temperature T_I, weighting factor c_T and other pre-specified parameters. Most parameters in the above equations are specified from external entities and are time invariant, e.g., g_1 and g_2. and c_T. However, the electricity price and the ideal temperature will changing with time according to the market and user's preference. It brings us the question how to efficiently setting the weighting factor c_T to evaluate the object function in Eq. (2.1) with changing

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parameters. The following section specifically describes an online estimation scheme to adjust the the weighting factor c_T.

IV. Weighting Factor Design via Online Estimation

The consumer satisfaction level coefficient c_T is generally changing based on the user preference and the price she/he is willing to pay. Therefore, $c_T(t)$ is a priori unknown and an adaptive scheme is required to specify this parameter at each time step. This problem can be formulated as an online estimation problem where the objective is to find the argument x that minimizes the cost function

$$f_t(x) = \frac{1}{2}||C_t - (P_t + x\Delta T(t))\,||_2^2 \text{ subject to } x \in \chi. \quad (4.7)$$

The variable C_t is the observation made at time t and is the value of actual cost function of Eq. (2.1) when the consumer has decided on $c_T(t)$ and the real price of the electricity of the day has been revealed. In addition, P_t and $\Delta T(t)$ are defined as

$$P_t = \sum_{s=t_0}^{t} c_p(s)\left(g_1 z_1(s) + g_2 z_2(s)\right) \quad (4.8)$$

$$\Delta T(t) = \sum_{s=t_0}^{t} |T(s) - T_I(s)|, \quad (4.9)$$

where the ideal temperature $T_I(s)$ can change over time based on the consumer preference and $z_1(s)$, $z_2(s)$, and $T(s)$ are determined from the above MILP algorithm. It is assumed that the value of cost function in (4.7) at time t is only revealed after $x(t)$ has been computed. In other words, $f_t(x)$ is allowed to change over time in an unpredictable manner due to modeling errors and uncertainties in the electricity price and user preference. The (sub)gradient of the estimation error (4.7),

$$\partial f_t(x) = -\Delta T(t)\left(C_t - (P_t + x\Delta T(t))\right), \quad (4.10)$$

is assumed to be a given determinant. Further, since the function $f_t(x)$ is convex on a compact domain, it is Lipschitz, i.e. there exists a positive constant L for which

$$|f_t(x) - f_t(y)| \le L\|x - y\| \;\forall\, x, y \in \chi. \quad (4.11)$$

The Lipschitz constant L can be found by observing that

$$\begin{aligned} &|f_t(x) - f_t(y)| \\ &= \frac{1}{2}\left|(x-y)^2 \Delta T(t)^2 - (C_t - P_t)\,\Delta T(t)\,(x-y)\right| \\ &\le \frac{1}{2}\left|(x-y)^2 \Delta T(t)^2\right| + |(C_t - P_t)\,\Delta T(t)\,(x-y)| \\ &\le \left(\frac{1}{2}\|\Delta T(t)\|_F \|x-y\|_2 + \|C_t - P_t\|_2\right) \times \\ &\quad \|\Delta T(t)\|_F \|x-y\|_2 \end{aligned}$$

We assume that the observation at time t is of the form $C_t(x) = a_t x + p_t$ for some $a_t \in (0, T_{\max} - T_{\min})$ and $p_t \in (0, p_{\max})$. In addition, $x \in (0, c_{T,\max})$ and $\Delta T \in (0, T_{\max} - T_{\min})$. Therefore,

$$\sup_{x\in\chi} \|C_t - P_t\|_2 \le (T_{\max} - T_{\min})\, c_{T,\max}.$$

and thus

$$L = \frac{3}{2} c_{T,\max}\,(T_{\max} - T_{\min})^2 \quad (4.12)$$

in (4.11). Note that L is a function of given system parameters, such as temperature range and the maximum value for the weighting factor.

In an off-line estimation problem, we have a noisy observation $C_t = P + x\Delta T + v_t$, where v_t is generally assumed to be (independent) white noise with covariance V_t at time t. In this case, the time-averaged optimal estimate [24] for (4.7) is

$$x^* = \frac{1}{N}\sum_{t=1}^{N}\left(\Delta T(t)^2 V_t^{-1}\right)^{-1}\left(\Delta T(t) V_t^{-1}(C_t - P_t)\right).$$

When $V_t = I$, the optimal estimate is $x^* = \frac{1}{N}\sum_{t=1}^{N} \Delta T(t)^{-1}(C_t - P_t)$. However, since the energy price and ideal temperature are time varying, the noise characteristics are not known and the off-line approach is not suitable for this problem. Therefore, we need an online scheme in which no assumption or knowledge of the statistical properties of the data are available. In the following proposed estimation algorithm, at time step t, the system estimates $x \in \chi$ and then an "oracle" announces the cost $f_t(x)$.

A. Online Estimation via Dual Averaging

In order to solve this estimation problem in an online setting, we exploited Nesterov's dual averaging algorithm [20], [21]. The Online Dual Averaging (ODA) algorithm is presented in Algorithm 1 where the state variable $x(t)$ and a dual variable $z(t)$ are updated sequentially:

$$d(t+1) = d(t) + \Delta(t),$$

where $\Delta(t) = \nabla f_t(x(t))$ is the (sub)gradient of the cost function. Then, $x(t+1) = \prod_{\chi}^{\psi}(d(t+1))$, where $\prod_{\chi}(\cdot)$ is a projection onto χ and is defined as

$$\Pi_{\chi}^{\psi}(d(t), \alpha(t)) = \arg\min_{x(t)\in\chi}\left\{\langle d(t), x\rangle + \frac{1}{\alpha(t)}\psi(x)\right\},$$

where $\alpha(t)$ is a non-increasing sequence of positive functions. In addition, $\psi(x) : \chi \to \mathbb{R}$ is a proximal function and is used to avoid wide oscillation in the projection step. Without loss of generality, ψ is assumed to be strongly convex with respect to $\|.\|$, $\psi \ge 0$ and $\psi(0) = 0$. Hence, we can select $\psi(x) = \frac{1}{k}\|x\|_2^2$ for a $k > 0$ which is bounded as $\psi(x) \le \frac{1}{k} c_{T,\max}^2$ in our problem setting.

The running average of the estimates $\{x(t)\}_{t=1}^{N}$, is also computed in Eq. (4.15) of the Algorithm (1).

for $t = 1$ *to* N **do**
  Adversary reveals $f_t(x(t))$
  Compute subgradient $\Delta(t) \in \partial f_t(x_t)$
  for *Each Agent* i **do**

$$d(t+1) = d(t) + \Delta(t) \tag{4.13}$$

$$x(t+1) = \prod_{\chi}^{\psi}(d(t+1), \alpha(t)) \tag{4.14}$$

$$\bar{x}(t+1) = \frac{1}{t+1}\sum_{s=1}^{t+1} x(s) \tag{4.15}$$

  end
end

Algorithm 1: Online Dual Averaging (ODA)

B. Convergence Analysis

Before proceeding to the convergence analysis of the online estimation algorithm, a few definitions are provided here. The goal of online algorithms is to ensure that the cumulative penalty algorithm pays because of its decisions on the cost sequence $\{f_t\}_{t=1}^N$ is small. In other words, time average of the difference between the total cost and the cost of the best fixed single decision x^* is sublinear as a function of N. This measure of the performance of learning algorithms is called *Regret* and is expressed as

$$R_N(x^*, x) = \sum_{t=1}^{N} \left(f_t(x(t)) - f_t(x^*)\right) \tag{4.16}$$

over $t = 1, 2, ..., N,$ iterations. If $\lim_{N\to\infty} R_N/N = 0$, the algorithm performs as well as the best fixed strategy independent of the uncertainties. In addition, The regret based on the *running average* of the estimates $\{x(t)\}_{t=1}^N$ is defined as

$$R_N(x^*, \bar{x}) = \sum_{t=1}^{N} \left(f_t(\bar{x}(t)) - f_t(x^*)\right), \tag{4.17}$$

where $\bar{x}(N) = \frac{1}{N}\sum_{t=1}^N x(t)$. Further discussion on online algorithms and their regret can be found in [25]–[27].

The following Lemma 4.1 can be found in [28], and so are presented here without proof.

Lemma 4.1: For any positive and non-increasing sequence $\alpha(t)$ and $x^* \in \chi$

$$\begin{aligned}\sum_{t=1}^{N} \langle \Delta(t), x(t) - x^*(t)\rangle \quad &\leq \quad \frac{1}{2}\sum_{t=1}^{N}\alpha(t-1)\|\Delta(t)\|_*^2 \\ &\quad + \frac{1}{\alpha(N)}\psi(x^*),\end{aligned}$$

where $||.||_*$ is the dual norm.[1]

Theorem 4.2: The sequence of $x(t)$ and $d(t)$ are generated by Eqs. (4.13) and (4.14) in Algorithm (1), with $\psi(x^*) \leq \frac{1}{k}c_{T,\max}^2$ and $\alpha(t) = q/\sqrt{t}$, where $k, q > 0$. Thus, we have

$$R_N(x^*, x) \quad \leq \left(qL^2 + \frac{1}{qk}c_{T,\max}^2\right)\sqrt{N} - \frac{L^2}{2}q \tag{4.18}$$

where L is given in (4.12).

Proof: The proof is similar to the regret analysis in [20], [21] and is presented here for the sake of completeness. An arbitrary fixed decision $x^* \in \chi$ and sequence $x(t)$ generated by Eq. (4.14) in Algorithm (1) are given. Since f_t are convex,

$$f_t(x(t)) - f_t(x^*) \leq \langle \Delta(t), x(t) - x^*\rangle \tag{4.19}$$

Thus, from the definition of regret in (4.16), it is verified that

$$R_N(x^*, x) \leq \sum_{t=1}^{N} \left(\langle \Delta(t), x(t) - x^*\rangle\right). \tag{4.20}$$

Therefore, using the bound in Lemma 4.1, we have

$$R_N(x^*, x) \leq \frac{1}{2}\sum_{t=1}^{N}\alpha(t-1)\|\Delta(t)\|_\star^2 + \frac{1}{\alpha(N)}\psi(x^*). \tag{4.21}$$

Note that convexity of f_t implies $\langle \Delta(t), x(t) - y(t)\rangle \leq f_t(x(t)) - f_t(y(t))$. Therefore, based on L-Lipschitz continuity of f_t, we have $||\Delta_i||_* \leq L$. Thus, using (4.21) the regret is further bounded as

$$R_N(x^*, x) \quad \leq \frac{L^2}{2}\sum_{t=1}^N \alpha(t-1) + \frac{1}{\alpha(N)}\psi(x^*), \tag{4.22}$$

and the theorem follows by applying the integral test on the first term[2] in (4.22) ■

The result shows the importance of the underlying system properties through parameter L as well as the "good" performance of the ODA algorithm through sublinear regret. Further, we can improve the regret bound by selecting appropriate values for $c_{T,\max}$, q, and k.

Next we exhibit a similar dependence on the parameters of the system for the regret analysis of the (temporal) running average estimates.

Corollary 4.3: The sequence of $\bar{x}(t)$ is generated by line 4.15 in Algorithm 1 with$\psi(x^*) \leq \frac{1}{k}c_{T,\max}^2$ and $\alpha(t) = q/\sqrt{t}$, where $k, q > 0$. Thus, we have

$$R_N(x^*, \bar{x}) \leq 2\left(qL^2 + \frac{1}{qk}c_{T,\max}^2\right)\sqrt{N}.$$

Proof: Since the cost function $f_t(x(t))$ is convex, $f_t(\bar{x}(t)) \leq \frac{1}{t}\sum_{s=1}^t f_t(x(s))$. Therefore, we have

$$f_t(\bar{x}(t)) - f_t(x^*) \leq \frac{1}{t}\sum_{s=1}^{t}\left(f_t(x(s)) - f_t(x^*)\right). \tag{4.23}$$

And given the definitions of regret in Eqs. (4.16) and (4.17), it comes that

$$R_N(x^*, \bar{x}) \leq \sum_{t=1}^{N}\left(\frac{1}{t}R_t(x^*, x)\right). \tag{4.24}$$

[1]Note that the dual norm of a vector x is defined as $||x||_* = \sup_{||y||=1} \langle x, y\rangle$.

[2]Note that $\sum_{t=1}^N \frac{q}{\sqrt{t}} \leq 2q\sqrt{N} - q$.

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Based on the regret bound in Eq. (4.18), we can further bound (4.24) as

$$\begin{aligned} R_N(x^*,\bar{x}) &\leq \sum_{t=1}^{N} \frac{1}{t}\left(\left(qL^2+\frac{1}{qk}c_{T,\max}^2\right)\sqrt{t}-\frac{L^2}{2}q\right) \\ &\leq 2\left(qL^2+\frac{1}{qk}c_{T,\max}^2\right)\sqrt{N}- \\ &\quad \left(qL^2+\frac{1}{qk}c_{T,\max}^2\right), \qquad (4.25) \end{aligned}$$

where the second inequality is based on the integral test and the corollary follows from (4.25). ∎

V. Simulation Example

In order to illustrate the feasibility of the proposed approach and the MILP model, a simulation example for a small scale cooling system is demonstrated in this section. We aim at validating the applicability and efficiency of the proposed approach for scheduling the operation of the cooling load. At the beginning of each day, we obtain the day-ahead predicted prices online from [8] and calculate the optimal schedules using the MILP algorithm with current ideal temperature setting and the updated weighting factor. At the end of that day, when the real-time price is published online, we adjust the weighting factor for the next day. This process is repeated as long as the operation schedule is required. Additional loads can be easily included in the current framework. The corresponding parameters of the cooling load used in the simulation are listed in Table I.

TABLE I
Parameters used in the cooling loads

$g_1(kw)$	$g_2(kw)$	$T_e(F)$	$T_{min}(F)$	$T_{max}(F)$
75	50	72	40	72
$\frac{Q_1}{C}$ (F/s)	$\frac{Q_2}{C}$ (F/s)	$\frac{1}{RC}$ (F/s)	Δt	$T_b(F)$
-3	-1.5	0.0225	2	T_I+5

During one day and night, the MILP algorithm generates optimal schedules for every hour with time step of two minutes based on the predicted price, the current ideal temperature setting from users and the designed weighting factor. In order to demonstrate the performance improvement by the proposed approach, we provide the temperature history, average power and cost using thermostat strategy without the optimal schedule. We assume the thermostat strategy always sets the weighting factor as $c_T = 0.002$ and the normal operation will switch on when the temperature is above $5F$ over the ideal one and switch off when it drops $5F$ below.

The simulation results of the temperature history in the first day with optimal and thermostat schedule are illustrated in Figure 2. As one can tell the average temperature with the optimal schedule is higher than the one with thermostat schedule. However, the optimized temperature is still bounded in a reasonable scale. This phenomenon indicates the regulation function of the proposed strategy when balancing between the energy cost and consumers' requests.

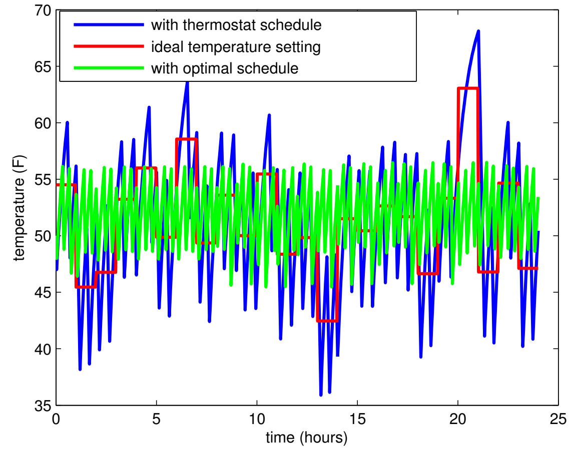

Fig. 2. Temperature history of chilling load with optimal and thermostat schedule for the first day.

We also provide the the cost comparison with optimal and thermostat schedule in Figure 3. The simulation results

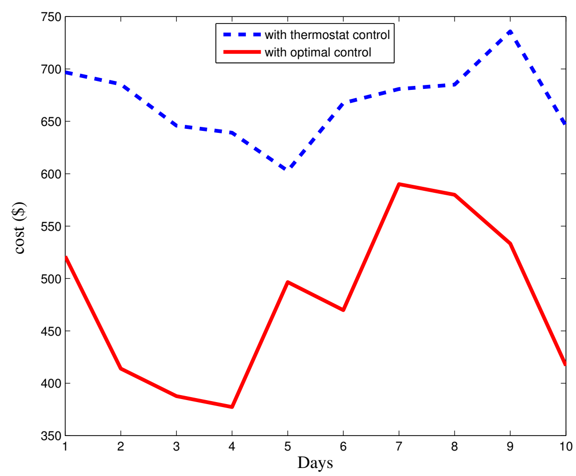

Fig. 3. Cost of each day with optimal and thermostat schedule.

confirm that the cost in the optimal scheduling scheme in any of the tenn days is lower than the corresponding ones using the thermostat schedule. However, for a cooling system with ten or hundred of similar loads, the saving of electricity cost by adopting the proposed approach is nontrivial.

In addition, the ODA algorithms was run on the described system for 10 days. The objective is to estimate a scalar $c_T \in (0, 0.01)$ with $q = \frac{1}{100}$, $k = 1$, and $(T_{\max}, T_{\min}) = (40, 72)$. Thus, $\chi = (0, 0.01)$, $c_{T,\max} = 0.01$, and $L = 15.36$. The ideal temperature is changing randomly every hour. Fig. (4) shows the good agreement of the estimated regret bound Eq. (4.18) and simulation results indicating that $R_N(x^*, x_1) = O(\sqrt{T})$.

VI. Conclusion

This paper presents an optimal power management strategy in cooling systems for real-time energy pricing scenario. Blending ideas from online dual averaging and optimization

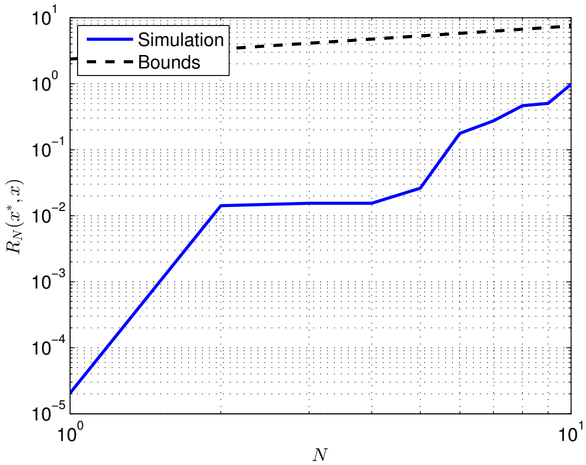

Fig. 4. Accuracy of bounds in Eq. (4.18).

with mixed integer linear programming is essential to control the cooling system operation and, meanwhile, adjust the weight factor as the real price of the electricity tariff is revealed. In this venue, the paper describes an optimization-based modeling technique for thermostatically controlled cooling load via mixed-integer linear programming (MILP), using the least number of binary variables. Simulation results show the efficacy of optimal power curtailment strategies and feasibility of integrating the proposed algorithm in future smart grid systems. Future research will consider more complex systems including various factors effecting ideal temperature setting and the operation of multiple loads in the cooling systems.

References

[1] H. Allcott, "Rethinking real-time electricity pricing," *Resource and Energy Economics*, vol. 33, no. 4, pp. 820–842, 2011.

[2] C. Aubin, D. Fougere, E. Husson, and M. Ivaldi, "Real-time pricing of electricity for residential customers: Econometric analysis of an experiment," *Journal of Applied Econometrics*, vol. 10, no. S1, pp. S171–S191, 1995.

[3] J.-C. Laurent, G. Desaulniers, R. P. Malhamé, and F. Soumis, "A column generation method for optimal load management via control of electric water heaters," *Power Systems, IEEE Transactions on*, vol. 10, no. 3, pp. 1389–1400, 1995.

[4] N. Lu, D. P. Chassin, and S. E. Widergren, "Modeling uncertainties in aggregated thermostatically controlled loads using a state queueing model," *Power Systems, IEEE Transactions on*, vol. 20, no. 2, pp. 725–733, 2005.

[5] C. Perfumo, E. Kofman, J. H. Braslavsky, and J. K. Ward, "Load management: Model-based control of aggregate power for populations of thermostatically controlled loads," *Energy Conversion and Management*, vol. 55, pp. 36–48, 2012.

[6] S. Katipamula and L. NING, "Evaluation of residential hvac control strategies for demand response programs," *ASHRAE transactions*, pp. 535–546, 2006.

[7] D. L. Ha, F. F. De Lamotte, and Q. H. Huynh, "Real-time dynamic multilevel optimization for demand-side load management," in *Industrial Engineering and Engineering Management, 2007 IEEE International Conference on*. IEEE, 2007, pp. 945–949.

[8] A. I. P. Co. (2010) Real-time pricing (rtp) for residential consumers. [Online]. Available: http://www.ameren.com/sites/aiu/MyHome/IlChoiceResidential/Pages/RealTimePricing.aspx

[9] C. E. Co. (2010) The comed residential real-time pricing program, guide to real-time pricing 2010. [Online]. Available: http://www.thewattspot.com/pdf/RRTPGuide200903.pdf

[10] M. A. Ortega-Vazquez and D. S. Kirschen, "Assessing the impact of wind power generation on operating costs," *Smart Grid, IEEE Transactions on*, vol. 1, no. 3, pp. 295–301, 2010.

[11] D. S. Callaway, "Tapping the energy storage potential in electric loads to deliver load following and regulation, with application to wind energy," *Energy Conversion and Management*, vol. 50, no. 5, pp. 1389–1400, 2009.

[12] K. Strunz and E. Kristina Brock, "Stochastic energy source access management: infrastructure-integrative modular plant for sustainable hydrogen-electric co-generation," *International journal of hydrogen energy*, vol. 31, no. 9, pp. 1129–1141, 2006.

[13] S. Abedi, A. Alimardani, G. Gharehpetian, G. Riahy, and S. Hosseinian, "A comprehensive method for optimal power management and design of hybrid res-based autonomous energy systems," *Renewable and Sustainable Energy Reviews*, vol. 16, no. 3, pp. 1577–1587, 2012.

[14] S. Ram, A. Nedic, and V. Veeravalli, "Incremental stochastic subgradient algorithms for convex optimization," *SIAM Journal on Optimization*, pp. 1–30, 2009.

[15] S. Sundhar Ram, A. Nedić, and V. V. Veeravalli, "Distributed Stochastic Subgradient Projection Algorithms for Convex Optimization," *Journal of Optimization Theory and Applications*, vol. 147, no. 3, pp. 516–545, 2010.

[16] A. Agarwal and J. Duchi, "Distributed delayed stochastic optimization," *IEEE Conference on Decision and Control*, pp. 5451–5452, 2012.

[17] I. Raykhel and D. Ventura, "Real-time automatic price prediction for ebay online trading." 2009.

[18] H. B. McMahan and A. Blum, "Online geometric optimization in the bandit setting against an adaptive adversary," in *Learning theory*. Springer, 2004, pp. 109–123.

[19] M. Zinkevich, "Online convex programming and generalized infinitesimal gradient ascent," 2003.

[20] L. Xiao, "Dual Averaging Methods for Regularized Stochastic Learning and Online Optimization," *Journal of Machine Learning Research*, vol. 11, pp. 2543–2596, 2010.

[21] Y. Nesterov, "Primal-dual subgradient methods for convex problems," *Mathematical Programming*, vol. 120, no. 1, pp. 221–259, Jun. 2007.

[22] S. Hosseini, A. Chapman, and M. Mesbahi, "Online Distributed Estimation via Adaptive Sensor Networks," *The IEEE Transactions on Control of Network Systems*, vol. submitted, 2014.

[23] I. CPLEX, "11.0 users manual," *ILOG SA, Gentilly, France*, 2007.

[24] L. Xiao, S. Boyd, and S. Lall, "A space-time diffusion scheme for peer-to-peer least-squares estimation," *Proceedings of the 5th international conference ...*, 2006.

[25] E. Hazan, "The Convex Optimization Approach to Regret Minimization," *Optimization for machine learning*, pp. 287–294, 2011.

[26] S. Bubeck, "Introduction to Online Optimization," *Lecture Notes*, 2011.

[27] S. Shalev-Shwartz, "Online learning and online convex optimization," *Foundations and Trends in Machine Learning*, vol. 4, pp. 107–194, 2012.

[28] J. C. Duchi, A. Agarwal, and M. J. Wainwright, "Dual averaging for distributed optimization: convergence analysis and network scaling," *IEEE Transactions on Automatic Control*, vol. 57, no. 3, pp. 592–606, 2012.