AN EXPLORATION OF FUEL OPTIMAL TWO-IMPULSE TRANSFERS TO CYCLERS IN THE EARTH-MOON SYSTEM

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This paper explores the optimum two-impulse transfer problem in a planar circular restricted three-body framework between a low Earth orbit and cycler orbits emphasizing the optimization strategy. Cyclers are those types of periodic orbits that meet both the Earth and the Moon periodically. A spacecraft on such trajectories are under the influence of both the Earth and the Moon gravitational fields. Cyclers have gained recent interest as baseline orbits for several Earth-Moon mission concepts. In this paper we show that a direct Lambert initial guess may not be adequate for these problems and propose a three-step optimization solver to improve the convergence toward an optimal solution. The first step consists of finding the feasible trajectories with a given transfer time. We employ Lambert’s problem to provide initial guess to optimize the error in arrival position. This includes the analysis of the liability of Lambert’s solution as an initial guess. Once the feasible trajectories are found, the impulse is a function of transfer time and the departure and arrival points’ phases. The second step consists of the optimization of impulse over transfer time which results in the minimum impulse transfer for fixed end points. The third is the study of the optimal solutions as the end points are varied. This study is done by use of the contours of optimal impulses in the space of departure and arrival points’ phases and the use of continuation methods. It is shown that Lambert does not work as initial guess for some regions in this space but that continuation methods can overcome this deficiency to same degree.

INTRODUCTION

The recent endeavor for Moon exploration has led to the exploration of several support architectures, such as telecommunications, navigation, and mapping missions. These mission concepts are supported by particular orbit structures such as periodic orbits and their manifolds. For example, the class of Lunar L2 halo family has been proposed as baseline for navigation and telecommunication architecture for the back side of the moon coverage. Another case, consist of cycler orbits which consist of large periodic orbits that meet both the Earth and Moon periodically. These orbits have been proposed as potential baseline orbits for transport, telecommunication and navigation architectures. In particular, Reference 2 proposes the use of such orbits for Earth-Moon constellations, whereby several spacecrafts at specific location on these orbits would provide a larger GPS-like navigation system for the Earth-Moon system. All these concepts thus require the controlled insertion of spacecraft on these periodic orbits. While halo orbits and the relative spacecraft insertion problem has been extensively studied (see for e.g. Reference 6–9), the cycler case has

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been much less studied. This paper focuses on transfers consist of a departure orbit in LEO and a
cycler as target orbit. The final orbit meets the Moon at a portion of its period. Addressing a general
approach for insertion on cycler orbits, this topic has not been given much attention.

**Literature Review**

As orbit transfers are the building blocks of any mission, many classes have been studied in detail
and a comprehensive survey is not possible here. Instead, we highlight the researches that have been
considered in this paper. First, Lambert’s problem, (i.e. the classic solution for near Earth orbit
transfers that consists of the determination of the conic arcs connecting two fixed position vectors
in a specified time) has been used as initial guess for solving the LEO to cycler transfer problem.
Lambert’s problem is formulated in the framework of Keplerian motion and does not include the
effect of Moon’s gravitational field (see for e.g. References 10 and 11 for general descriptions).
Many Lambert solver have been derived and we considered the one of Oldenhuis.12 Of interest
is the minimum $\Delta V$ Lambert problem, for which only numerical approaches are available.13,14
In the following work we use continuation in the transfer time to obtain minimum $\Delta V$ Lambert solutions.15

The case of transfers between low Earth and low Moon orbits have been studied by a variety of
approaches, such as minimal energy path, patched conic method, and gravitational capture.16 These
methods take into account the Earth’s and Moon’s gravitational fields and the mutual interactions
between them. The estimated transfer path is between a low earth orbit and a lunar parking or-
bit. In the gravitational capture method, the spacecraft approaches one of the primary bodies and
orbits temporarily around it. Minimal energy path and patched conic method are based on the two-
body Earth probe and Moon probe problems which have errors due to effect of Earth and Moon
gravitational fields in each other’s probe.

Low energy Earth-Moon transfers have also been studied in the circular restricted three body
problem (CRTBP) model using libration point dynamics (see for e.g. References17–20). Periodic
orbits in the CRTBP could be characterized into families. The invariant manifolds theory is useful
in finding an optimum transfer between Earth and Moon. Based on this theory and libration points,
optimal transfers have been found between particular classes of periodic orbits such as unstable
ones.

The class of transfers from LEO to cyclers has been much less studied, and this paper provides
a first step in a systematic study of this problem. Specifically, using a two-impulse transfer, a
spacecraft is sent from a point near the Earth to an orbit which encounters both Earth and Moon,
called a cycler. While an end result is the ability to quickly find the optimal transfer between a LEO
and an arbitrary cycler orbits, this paper focuses on mapping the set of feasible transfers between
the initial and target orbits, and better understanding the dependence of the transfer costs with the
boundary conditions. In addition, the use of a staged approach to generating transfers solutions
allows us to explore the convergence issues when using a direct optimization (Matlab fmincon
function) from a Lambert initial guess compared to a homotopy method.15

**Outline**

This paper is organized as follows; the first section reviews the necessary background on the
dynamical models that are used in optimization problem and generating transfer trajectories. Next,
the optimization problem is expressed in a proper form followed by the optimization strategy. This includes the schematic computation of good initial guess and the differential correction to feasible transfers between two position vectors. A feasible transfer is found by optimizing the error in final position vector. By varying transfer time, we find the feasible transfers and develop a plot that shows the total $\Delta V$ as a function of transfer time for an arbitrary case. Another optimization is needed in order to find the minimum $\Delta V$. Since, we acquire enough information for different cases, we generate the contour of minimum $\Delta V$ in the plane of departure and arrival points phases. This contour helps us to explore the transfers and their inherent difficulties in presence of the Moon.

**DYNAMICAL MODEL AND BOUNDARY CONDITIONS**

In order to define a transfer, a dynamics and boundary conditions must be defined. The (CRTBP) is used as the baseline dynamical model in this study and a brief review of this well-known model\(^\text{21}\) is given first to set up the notations.

**The Circular Restricted Three Body Problem**

The CRTBP addresses the motion of a particle $P$ of negligible mass moving under the gravitational influence of masses $m_E$ and $m_M$, referred to as the primary masses. The mass $m_E$ represents the mass of the Earth and $m_M$ represents the mass of the Moon, and we are concerned with the motion of $P$, a spacecraft of much smaller mass. Assume that the Earth and the Moon have circular orbits about their common center of mass. The spacecraft $P$ is free to move in the plane defined by the circular orbits of the primaries, but cannot affect their motion\(^\text{21}\) (see Figure 1).

![Figure 1. Geometry of the CRTBP in the synodic frame](image)

A convention has emerged on the units and coordinates frames for the restricted problem since this problem has been studied\(^\text{22}\). The length unit is normalized by the distance between the two primaries. The mass is normalized by the sum of the masses of the Earth and Moon, and the time is normalized by the orbital period of them.

\[
\mu = \frac{m_M}{m_E + m_M} = 0.0121505 \quad \mu_M = \mu \quad \mu_E = 1 - \mu \quad (1)
\]

\[
d = \frac{d}{L} \quad \dot{t} = \frac{t}{T} 2\pi \quad (2)
\]
Where $T$ is the orbital period of the Earth and Moon (27.3453 days), and $L$ is the distance between them (385000 km). In order to reduce the dynamics to autonomous set of ordinary differential equations (ODE), the CRTBP is usually formulated in the synodic reference frame. The synodic reference frame rotates with the primaries about their barycenter and its origin is at the center of mass of the Earth-Moon system. The $x$-axis is directed from the Earth to the Moon, and the $y$-axis is orthogonal to the $x$-axis in the primary plane of motion. The $z$-axis is found by the right hand rule.

Using the equation of motion of the spacecraft in the CRTBP we can find the following relation between the state and vector filed of the system given in Eq. (3) and Eq. (4).

$$X = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z}]^T$$  \hspace{1cm} (3)

$$F(X) = \frac{dX}{dt} = [\dot{x} \quad \dot{y} \quad \dot{z} \quad 2\dot{y} - U_x \quad -2\dot{x} - U_y \quad -U_z]^T$$  \hspace{1cm} (4)

Where $U$ is the augmented or effective potential and the subscripts in Eq. (4) denote its partial derivatives.

$$U = -\frac{1}{2}(x^2 + y^2) - \frac{\mu_E}{r_1} - \frac{\mu_M}{r_2} - \frac{1}{2}\mu_E\mu_M$$  \hspace{1cm} (5)

$r_1$ and $r_2$ are the distances of the spacecraft from the Earth and the Moon, given in the synodic frame components.

$$r_1 = \sqrt{(x + \mu_M)^2 + y^2 + z^2} \hspace{1cm} r_2 = \sqrt{(x - \mu_E)^2 + y^2 + z^2}$$  \hspace{1cm} (6)

**Initial and Target Orbits**

With the dynamical model defined, a transfer boundary condition need to be defined. In this work we consider transfers between particular periodic orbits. Periodic orbits corresponds to a special type of solution to the dynamics (Eq. (4)) which repeat themselves after a specific time span, the orbit period $T$. An interesting property of such orbit is the generic parameterizations of a spacecraft location as an angle (phase). More specifically, The phase of each point is defined by Eq. (7) and is proportional to the amount of time spent to arrive that point from the given initial conditions.

$$\theta = \frac{2\pi}{T}.t$$  \hspace{1cm} (7)

That is, the phase defines a continuous mapping from the time axis to the circle. This geometry is depicted on Figure 2 (left). The departure orbit in this paper is chosen as representative of a parking orbit around the Earth, that is a 600 km altitude, direct circular periodic orbit, as shown in Figure 2 (right).

The target orbits are chosen as sample cyclers.\textsuperscript{5} As mentioned in the introduction, cyclers are those types of periodic orbits that meet both the Earth and the Moon periodically. Initial conditions from Casoliva et al.\textsuperscript{5} have been used to select sample cycler orbits for the following numerical study (see Table 1). As can be seen these orbits encircle both the Earth and the Moon in the synodic reference frame of the CRTBP (see Figure 3).
Figure 2. Left: Phase definition along a periodic orbit. The Earth is located at \((-\mu, 0, 0)\) and the Moon is located at \((1 - \mu, 0, 0)\). While quite different from the LEO, departure periodic orbit shown on the Right plot, both orbit mapped when viewed from a phasing viewpoint can both be seen as circle, with the phase varying from 0 to \(2\pi\).

Table 1. Initial conditions for sample cycler orbits (normalized units)

<table>
<thead>
<tr>
<th>Designation</th>
<th>T</th>
<th>(x(0))</th>
<th>(y(0))</th>
<th>(z(0))</th>
<th>(u(0))</th>
<th>(v(0))</th>
<th>(w(0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>12.566371</td>
<td>2.471726</td>
<td>0.0</td>
<td>0.0</td>
<td>0.378317</td>
<td>-2.280067</td>
<td>0.0</td>
</tr>
<tr>
<td>(b)</td>
<td>12.580503</td>
<td>1.674044</td>
<td>0.0</td>
<td>-1.584540</td>
<td>3.869731e-7</td>
<td>-2.1018505</td>
<td>2.194117e-8</td>
</tr>
</tbody>
</table>

The table components are represented in the normalized synodic coordinate frame.

Figure 3. Periodic Orbits in Synodic frame. The Earth is located at \((-\mu, 0, 0)\) and the Moon is located at \((1 - \mu, 0, 0)\). Left: The cycler is in Earth-Moon orbital plane (planar case). Right: The projection of a cycler which is in 3D space on the Earth-Moon orbital plane.
PROBLEM STATEMENT AND SOLUTION STRATEGY

The problem of analyzing 2-impulse fuel optimal transfers between a LEO and a cycler is formulated as minimization problem, where the total $\Delta V$ is used as the cost function and the state vector $\vec{r}(t)$ is constrained to satisfy the circular restricted three body dynamics (denoted succinctly as $\ddot{\vec{r}} = F(\vec{r}, \dot{\vec{r}})$) and boundary conditions. Since the transfer trajectories are produced by time integration of the equations of motion in the CRTBP model, the total impulse ($\Delta V$) for the given end points is only a function of transfer time. Therefore, the problem is to optimize the total $\Delta V$ over transfer time subject to two constraints. These constraints are the boundary conditions arise from the end points’ position vectors. The mathematical format of the problem is depicted in Eq. (8) and the boundary conditions are given in Eq. (9).

$$\min_{\Delta t} \Delta V(\Delta t)$$

such that

$$\begin{cases} \vec{r}(t_0) = \vec{r}_1(\theta_1) \\ \vec{r}(t_1) = \vec{r}_2(\theta_2) \end{cases}$$

(9)

Where the $\theta_1$ and $\theta_2$ represent the phases on the initial and target periodic orbits.

Two strategies have been experimented to find the fuel optimal transfers. The first strategy consists of attempting to solve the above optimization problem (see Eq. (8)) with a generic optimization routine (Matlab fmincon function), while the second approach consists of using continuation (homotopy) to smoothly deform Lambert solution to the desired optimum. Note that in both approaches, the optimal transfer and its period are found at a specific departure and arrival phase before analyzing the variation of the solution with these boundary conditions (the orbit being kept always fixed). This optimal solution is used as the initial guess for the next close boundary condition in phase space.

The use of generic optimization package is easier to implement and does not require a specialized formulation, as opposed to the continuation approach. However, the direct approach appeared as more dependent on the choice of initial guess than the alternative approach and could not converge directly and a better initial guess than Lambert was required to converge. We thus used the optimization routine to generate a better initial guess by correcting Lambert to satisfy the boundary condition (feasible trajectory problem) before attempting to solve for the optimal $\Delta V$. However, in each case, solution to Lambert’s problem is used to start the both optimization and continuation approaches, and this problem is thus reviewed next.

**Lambert’s problem as initial guess**

Lambert’s problem consists of finding transfer orbits that connect two position vectors in a given transfer time in a 2-body field, that is to solve the following two-point boundary value problem:

$$\begin{cases} \ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} \\ \vec{r}(t_0) = \vec{r}_0 \\ \vec{r}(t_1) = \vec{r}_1 \end{cases}$$

(10)

This problem has been addressed extensively in the literature and there have been many contributions to its solution (see for e.g. References 10, 11, 23, 24). Lambert’s original formulation provides...
a way to determine the minimum energy transfer subject to the stated boundary conditions. Given
that a 2-body orbit can be parameterized by orbital elements, we have:

\[ r(a, e, f_1) = r_1; \quad r(a, e, f_2) = r_2 \quad (11) \]

and:

\[ f_2 = f_1 + \theta \quad (12) \]

\[ E_2 - E_1 - e \cdot \sin(E_2) + e \cdot \sin(E_1) = \Delta \tau \quad (13) \]

In the above equations (Eq. (11), (12), and (13)), \( r_1 \) and \( r_2 \) refer to the radial distance of \( P \) from the attracting center (see Figure 4). The angles \( f_1 \) and \( f_2 \) are the true anomaly of \( P_1 \) and \( P_2 \) on the transfer orbit, and \( E_1 \) and \( E_2 \) stand for the mean anomaly of \( P_1 \) and \( P_2 \), respectively. In addition, \( \Delta \tau \) is the transfer time between \( P_1 \) and \( P_2 \) which are departure and arrival points. In the two-body

![Figure 4. Transfer orbit geometry for Lambert’s problem](image)

problem, Lambert’s theorem states that the orbital transfer time depends only on the semimajor axis, the sum of the distances of the initial and final points, and the length of the chord between these points. Its solution is the conic orbits depicted in Figure 4. Lambert’s problem has two solutions for a given transfer time, and there are almost always two ellipses that satisfy the boundary conditions*. On one of the transfers, spacecraft travels to the arrival point along the direct trajectory which is the short way and on the other transfer, it travels along the long way, called indirect trajectory. The direct trajectory is considered for the following simulations.

In this paper, the solver developed by Oldenhuis12† (which is based on two robust separate algorithms24,25) is used in the direct optimization approach, while the homotopy method introduced in Reference15 is used for the continuation approach. In that case a linear guess between the boundary conditions is used to start a continuation in transfer time (in a two-body dynamics) to reach the desired solution.

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*In a two-body problem the total energy of the system depends only on the semimajor axis of the orbit \( (\varepsilon = -\frac{\mu}{a}) \). For a circular orbit the energy is zero and for an elliptic and a hyperbolic orbit, energy is negative and positive respectively. The minimum energy ellipse is an orbit which has the minimum required energy that transfers the spacecraft between given end points.

†http://www.esa.int/gsp/ACT/inf/op/globopt.htm
The evaluation of Lambert’s initial guess when applied to the three-body dynamics has been done by computing the off track error consists of the difference between the desired boundary condition and Lambert’s initial guess. Figure 5 shows this off track error at the final position for the transfer times of 10 days (see Figure 7(d)) and 15 days (see Figure ??). The off track error is mainly depends on the arrival point phase in those cases.

Figure 5. Contour plots of the off track error at arrival point when applying Lambert’s initial case in the CR3BP for the planar cycler considered. Left: 10 days case; Right: 15 days case. The error is measure in normalized units.

In order to study the characteristics of transfers, several points of these phase spaces have been selected along specific departure phase (see Table 2).

<table>
<thead>
<tr>
<th>θ₁</th>
<th>Figure(a)</th>
<th>Figure(b)</th>
<th>Figure(c)</th>
<th>Figure(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 rad</td>
<td>θ₂ = 0.38492 rad</td>
<td>θ₂ = 0.93939 rad</td>
<td>θ₂ = 4.2024 rad</td>
<td>θ₂ = 4.7903 rad</td>
</tr>
<tr>
<td>3.6 rad</td>
<td>θ₂ = 0.38492 rad</td>
<td>θ₂ = 0.93939 rad</td>
<td>θ₂ = 4.2024 rad</td>
<td>θ₂ = 4.7903 rad</td>
</tr>
</tbody>
</table>

These Figures( 6 and 7) shows that when the space craft has a close fly-by of the Moon the off track error is large and lead to a failure to converge on a feasible trajectory using fmincon. Figure 5 is a contour plot of this off-track error for a larger set of orbits, showing the difficulty in using Lambert problem for a significant portion of trajectories.

Feasible Trajectory Problem

While Lambert is used to start our approaches, the correction of Lambert initial guess to a feasible trajectory has been found to be necessary before attempting to solve for the optimal ∆V when using fmincon. The feasible trajectory problem is then formulated as a forward shooting method: suppose we have the initial impulse velocity (∆V₁) which inserts the spacecraft on the transfer orbit at departure point. Integrating the equations of motion until it reaches the specified transfer time (∆t), the final position vector found is then compared with the desired boundary condition. The
Figure 6. Sample Trajectories along $\Theta_1 = 0 rad$ for 10 days transfer time
Figure 7. Sample Trajectories along $\Theta_1 = 3.6\, \text{rad}$ for 10 days transfer time
distance between these two positions is used as the cost function of the feasible trajectory problem:

$$\min_{\Delta V_1} \left\| \mathbf{r}_2(\theta_2) - \mathbf{r}_1(\Delta V_1) \right\| \quad \text{subject to} \quad \begin{cases} \mathbf{r}(0) = \mathbf{r}_1(\theta_1) \\ \Delta t \text{ is given} \end{cases}$$

(14)

Note that the initial guess for $\Delta V_1$ required by the optimization routine to start attempting to solve this minimization problem is provided by Lambert solution. In essence, solving the minimization problem represent solving a Lambert problem in the CR3BP. Once this feasibility problem is solved for an arbitrary $\Delta t$ a line search over $\Delta V$ allows for the solution to Eq. (8).

In the continuation approach, the deformation of the dynamics to smoothly vary from a two-body system to the CR3BP is used to generate feasible trajectories. A continuation in the transfer time is then used, to locate the extremal $\Delta V$. The next section described the results obtained on this feasible trajectory problem.

**NUMERICAL RESULTS**

This section describes the results obtained for the above optimization strategies in the case of the two cyclers presented in Figure 3. First the results related to the feasible trajectory computation are presented, before the optimization over transfer time is considered.

**Feasible Trajectories**

A feasible transfer trajectory between a low Earth orbit (LEO) at 600 km altitude and an arbitrarily cycler is depicted in Figure 8. This feasible transfer has a specified transfer time. If we do the above optimization problem for several transfer time, we can find $\Delta V_1$ of feasible transfers as a function of transfer time. This relation is also depicted on the right hand side of Figure 8. Once a feasible trajectory can be computed, the minimum $\Delta V$ problem (see Eq. (8)) is reduced, in essence, to a one dimensional line search over $\Delta V$.

![Feasible trajectory and its ΔV versus transfer time](image)

**Figure 8.** A feasible trajectory and its $\Delta V$ versus transfer time

In order to capture a larger range of orbits, contour plots of the feasible trajectories $\Delta V$ as a function of the boundary condition phases has been considered for a fixed transfer time. In the
direct optimization approach to Lambert, it is found that \texttt{fmincon} does not succeed to converge where the off track error of the Lambert initial guess is greater than 400 km. As was noted in the previous section, these transfers correspond to cases that encounter the Moon. For the other cases, the Lambert solution stay away from the Moon all along the transfer and convergence can be obtained. In fact, as shown in in Figs. 9 and 10, the difference in \( \Delta V \) costs between the Lambert’s solution and the differentially corrected feasible trajectory remains small in the cases of convergence.

The results shown on Figs. 9 and 10 were in fact computed with the continuation approach. Non-convergence trajectories (shown as white or dark blue regions) are also encountered with this approach, but not necessarily for the same phase angle values. In effect, non-convergence appears when impact with the Moon are encountered during the continuation process. However, the continuation method appears as more robust than the direct numerical optimization of the off track error using \texttt{fmincon}. Several days of numerical simulation on a cluster were required to partially complete a map with the matlab optimization function, while the numerical continuation performed in AUTO-07p\textsuperscript{26} required only 6 to 8 hours on a laptop.

![Figure 10. \( \Delta V \) maps for Lambert problem (Left column) and feasible trajectories (Right column) for 10 days (Top row) and 15 days transfers (Bottom row) for the second cycler considered (spatial problem). The maps have been computed with the continuation method for a grid of 90\times90 points.](image)

Interesting features to observe from these plots is the discontinuity of the results at the Moon.
encounter region (non-convergence points) where large $\Delta V$ change to much lower value depending on the way the Moon is approached. Also, the extent of non-convergence (and thus encounter of a Moon close approach during the continuation process) increases with the increase in transfer time.

In the case of the 3-dimensional cycler orbit, the non-convergence regions are much smaller and Lambert’s initial guess provides here a good initial guess for most of the trajectories. The near periodicity of the non-convergence regions reflects the topology of the cycler orbits which present two moon encounters. Further investigation of these features will be investigated in future work.

**OPTIMAL $\Delta V$ PROBLEM**

As mentioned in the previous section, once a feasible trajectory is found, a line search or 1-dimensional continuation in the transfer time is used to locate extrema of the $\Delta V$ along the feasible transfer family with fixed boundary phases. Performing this line search for a grid of initial phases
leads a $\Delta V$ contour map, as shown in Fig. 11. The non-convergence regions as depicted in Figures 9 and 10 have very high $\Delta V$. The feasible trajectories in those regions have very low transfer time because the transfer trajectories do not encounter the Moon. These transfer orbits are straight lines and the $\Delta V$ required to reach the target point are very high. In addition, the non-convergence regions reflects the the regions where no feasible trajectory has found for the given transfer times span.

![DV map for the minimum DV solution](image1)

**Figure 11.** Minimum (Left) and maximum (Right) $\Delta V$ maps for transfers between 1 days and 15 days transfers for the planar cycler investigated, as a function of the boundary orbit phases. Units are non-dimensional.

**CONCLUSION**

This paper described methods to construct optimal two impulse orbit transfers in CRTBP model using two steps approach. First, an optimization over the arrival position vectors error has been applied in order to find the admissible transfers. Lambert’s problem has been implemented in the optimization problem as an initial guess. This technique may be used to find the total impulse velocity as a function of transfer time and end points’ phases. Then a set of feasible transfers is found and minimum $\Delta V$ problem is reduced to a one parameter optimization problem. The minimum $\Delta V$ depends only on departure and arrival phases. The 2D map summarizing this dependence provides an interesting product for space mission design. Lambert i.e. does not seem appropriate for initial guess since the transfers at some points in 2D map have large errors in arrival position vectors. This error would likely require a prompt correction in optimization problem. While Initial guess has significant effect on the optimization problem convergence and solution, it is necessary to improve the initial guess. A good approach to modify the initial guess might be the minimum $\Delta V$ Lambert’s problem. This method probably would accelerate the optimization convergence and is worth to study its effect on cyclers’ transfer. This method may not be sufficient since it is in two body framework. Another good approach is Modified Lambert’s problem which may be more effective than the minimum $\Delta V$ Lambert’s problem. In this method, we encounter the effect of Moon gravitational field in Lambert’s problem and try to find a transfer orbit that connects to given position in space at a specified time interval. The authors are currently investigating this idea.
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